Class 11 - Important Formulas

Chapter 7 - System of Particles and Rotational Motion

Mechanics of system of Particles

S.No.	Term	Description
1	**Centre of mass**	It is that point where entire mass of the system is imagined to be concentrated, for consideration of its translational motion.
2	**position vector of centre of mass**	$\mathbf{R}_{cm}=\sum \mathbf{r}_i M_i/\sum M_i$ where $\mathbf{r}_i$ is the coordinate of element i and M_i is mass of element i
3	**In coordinate system**	$x_{cm}=\sum x_i M_i/\sum M_i$ $y_{cm}=\sum y_i M_i/\sum M_i$ $z_{cm}=\sum z_i M_i/\sum M_i$
4	**Velocity of CM**	$\mathbf{v}_{CM}=\sum \mathbf{v}_i M_i/\sum M_i$ The total momentum of a system of particles is equal the total mass times the velocity of the centre of mass
5	**Force**	When Newton's second law of motion is applied to the system of particles we find $F_{tot}=Ma_{CM}$ with $a_{CM}=d^2R_{CM}/dt^2$ Thus centre of mass of the system moves as if all the mass of the system were concentrated at the centre of mass and external force were applied to that point.
6	**Momentum conservation in COM motion**	$\mathbf{P}=M\mathbf{v}_{CM}$ which means that total linear momentum of system of particles is equal to the product of the total mass of the system and the velocity of its centre of mass.

Rigid body Dynamics

S.No.	Term	Description
1	**Angular Displacement**	-When a rigid body rotates about a fixed axis, the angular displacement is the angle $\Delta\theta$ swept out by a line passing through any point on the body and intersecting the axis of rotation perpendicularly -It can be positive (counter clockwise) or negative (clockwise). -Analogous to a component of the displacement vector. -SI unit: radian (rad). Other units: degree, revolution.
2	**Angular velocity**	-Average angular velocity, is equal to $\Delta\theta/\Delta t$. Instantaneous Angular Velocity $\boldsymbol{\omega}=d\theta/dt$ -Angular velocity can be positive or negative
		-It is a vector quantity and direction is perpendicular to the plane of rotation -Angular velocity of a particle is different about different points -Angular velocity of all the particles of a rigid body is same about a point.
3	**Angular Acceleration**	Average angular acceleration= $\Delta\boldsymbol{\omega}/\Delta t$ Instantaneous Angular Acceleration $\alpha=d\boldsymbol{\omega}/dt$
4	**Vector Nature of Angular Variables**	-The direction of an angular variable vector is along the axis. - positive direction defined by the right hand rule. - Usually we will stay with a fixed axis and thus can work in the scalar form. -angular displacement cannot be added like vectors. Angular velocity and acceleration are vectors

5	**Kinematics of rotational Motion**	$\boldsymbol{\omega}=\boldsymbol{\omega_0} + \boldsymbol{\alpha}t$ $\boldsymbol{\theta}=\boldsymbol{\omega_0}t+1/2\boldsymbol{\alpha}t^2$ $\boldsymbol{\omega}.\boldsymbol{\omega}=\boldsymbol{\omega_0}.\boldsymbol{\omega_0} + 2\ \boldsymbol{\alpha}.\boldsymbol{\theta}$; Also $\alpha=d\omega/dt=\omega(d\omega/d\theta)$
6	**Relation Between Linear and angular variables**	$\mathbf{v}=\boldsymbol{\omega}X\mathbf{r}$ Where r is vector joining the location of the particle and point about which angular velocity is being computed $\mathbf{a}=\boldsymbol{\alpha}X\mathbf{r}$
7	**Moment of Inertia**	Rotational Inertia (Moment of Inertia) about a Fixed Axis For a group of particles, $I = \Sigma mr^2$ For a continuous body, $I = \int r^2dm$ For a body of uniform density $I = \rho\int r^2dV$
8	**Parallel Axis Therom**	$I_{xx}=I_{cc}+ Md^2$ Where I_{cc} is the moment of inertia about the centre of mass
9	**Perpendicular Axis Therom**	$I_{xx}+I_{yy}=I_{zz}$ It is valid for plane laminas only.
10	**Torque**	$\boldsymbol{\tau}=\mathbf{r}X\mathbf{F}$ also $\tau=I\alpha$ where α is angular acceleration of the body.
11	**Rotational Kinetic Energy**	$KE=(1/2)I\omega^2$ where ω is angular acceleration of the body
12	**Rotational Work Done**	-If a force is acting on a rotating object for a tangential displacement of $s = r\theta$ (with θ being the angular displacement and r being the radius) and during which the force keeps a tangential direction and a constant magnitude of F, and with a constant perpendicular distance r (the lever arm) to the axis of rotation, then the work done by the force is: $W=\tau\theta$ -W is positive if the torque τ and θ are of the same direction, otherwise, it can be negative.
13	**Power**	$P =dW/dt=\tau\omega$
14	**Angular Momentum**	$\mathbf{L}=\mathbf{r}X\mathbf{p}$ $=\mathbf{r}X(m\mathbf{v})$ $=m(\mathbf{r}X\mathbf{v})$ For a rigid body rotating about a fixed axis $L=I\omega$ and $d\mathbf{L}/dt=\boldsymbol{\tau}$ if $\tau=0$ and L is constant For rigid body having both translational motion and rotational motion $\mathbf{L}=\mathbf{L_1}+\mathbf{L_2}$ $\mathbf{L_1}$ is the angular momentum of Centre mass about an stationary axis $\mathbf{L_2}$ is the angular momentum of the rigid body about Centre of mass.
15	**Law of Conservation On Angular Momentum**	If the external torque is zero on the system then Angular momentum remains contants $d\mathbf{L}/dt=\boldsymbol{\tau_{ext}}$ if $\boldsymbol{\tau_{ext}}=0$ then $d\mathbf{L}/dt=0$
16	**Equilibrium of a rigid body**	$\mathbf{F_{net}}=0$ and $\boldsymbol{\tau_{ext}}=0$
17	**Angular Impulse**	$\int\tau dt$ term is called angular impulse. It is basically the change in angular momentum
18	**Pure rolling motion of sphere/cylinder/disc**	-Relative velocity of the point of contact between the body and platform is zero -Friction is responsible for pure rolling motion -If friction is non dissipative in nature $E = (1/2)mv_{cm}^2+(1/2)I\omega^2+mgh$