Important Questions for Class 9

Maths

Chapter 12 - Heron's Formula

Section A

1. An isosceles right triangle has an area $8\,cm^2$. The length of its hypotenuse is

1. $\sqrt{16}\,cm$

2. $\sqrt{48}\,cm$

3. $\sqrt{32}\,cm$

4. $\sqrt{24}\,cm$

Ans: Height of triangle $=h$

As the triangle is isosceles,

height $=\mathbf{h}$

Area of triangle $=8\,cm^2$

$\Rightarrow \frac{1}{2}\times$ Base $\times$ Height $=8$

$\Rightarrow \frac{1}{2}\times h\times h=8$

$\Rightarrow h^2=16$

$\Rightarrow h=4\,cm$

Base =Height $=4\,cm$

Since the triangle is right angled,

Hypotenuse $^2=$ Base $^2+$ Height ^{2}

$\Rightarrow$ Hypotenuse $^2 = 4^2 + 4^2$

$\Rightarrow$ Hypotenuse $^2 = 32$

$\Rightarrow$ Hypotenuse $= \sqrt{32}$

Therefore, Options C is the correct answer.

2. The sides of a triangle are $35\text{ cm}, 54\text{ cm}$, and 61 cm, respectively. The length of its longest altitude is

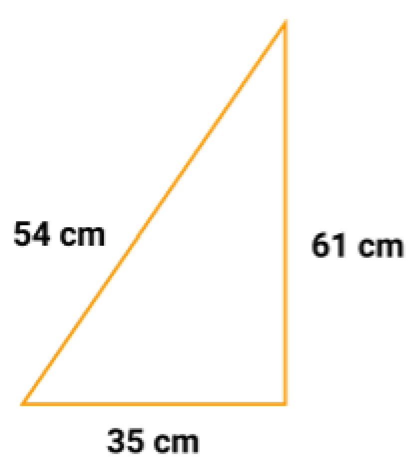
1. $26\sqrt{5}\text{cm}$

2. 28 cm

3. $10\sqrt{5}\text{cm}$

4. $24\sqrt{5}\text{ cm}$

Ans: Semi-perimeter of a triangle,



$$s = \frac{a+b+c}{2}$$

$$= \frac{35+54+61}{2}$$

$= 75cm$

Area A

$A = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{75(75-35)(75-54)(75-61)}$

$= 420\sqrt{5}\ \text{cm}^2$

Area of the triangle is also given as $A = \frac{1}{2} \times \mathbf{a} \times \mathbf{h}$

Where, h is the longest altitude.

Therefore, $\frac{1}{2} \times a \times h = 420\sqrt{5}$

$\Rightarrow h = \frac{420 \times 2 \times \sqrt{5}}{a}$

$\Rightarrow h = \frac{420 \times 2 \times \sqrt{5}}{35}$

Hence, the length of the altitude $h = 24\sqrt{5}\ \text{cm}$

3. The sides of a triangle are $56\ \text{cm}, 60\ \text{cm}.$ and $52\ \text{cm}$. long. The area of the triangle is.

1. $4311\ \text{cm}^2$

2. $4322\ \text{cm}^2$

3. $2392\ \text{cm}^2$

4. None of these

Ans: The three sides of a triangle are $a = 56\ \text{cm}, b = 60\ \text{cm}$ and $c = 52\ \text{cm}$. Then, semi-perimeter of a triangle,

$s=\frac{a+b+c}{2}=\frac{56+60+52}{2}=\frac{168}{2}=84\text{ cm}$

Area of a triangle

= $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{84(84-56)(84-60)(84-52)}$

= $\sqrt{4\times7\times3\times4\times7\times4\times2\times3\times4\times4\times2}$

= $\sqrt{(4)^6\times(7)^2\times(3)^2}$

= $(4)^3\times7\times3=1344\text{ cm}^2$

The area of triangle is 1344 cm^2.

Therefore, the option (4) is the correct answer.

4. The area of an equilateral triangle is $16\sqrt{3}\text{ m}^2$. Its perimeter is

1. 24 m

2. 12 m

3. 306 m

4. 48m

Ans: Let the side of the equilateral triangle be am

Now, area of equilateral $\Delta=\frac{\sqrt{3}}{4}(\text{ side })^2$

$\Rightarrow 16\sqrt{3}=\frac{\sqrt{3}}{4}(\text{a})^2$

$\Rightarrow \text{a}^2=\frac{16\sqrt{3}\times4}{\sqrt{3}}=64$

$\Rightarrow a = \sqrt{64}$

$= 8\text{ m}$

Substitute the value of a

Perimeter of equilateral $\Delta = 3a = 3 \times 8$.

$= 24\text{ m}$

Therefore, option (4) is the correct answer.

5. The perimeter of a triangle is 30 cm. Its sides are in the ratio $1:\ 3:\ 2$, then its smallest side is.

1. 15 cm

2. 5cm

3. 1 cm

4. 10 cm.

Ans: Perimeter of triangle $= 30\text{ cm}$

Ratio of its sides are $= 1:3:2$

sides are $x, 3x, 2x$

$\Rightarrow x + 3x + 2x = 30\text{ cm}$

$\Rightarrow 6x = 30$

$\Rightarrow x = 5\text{ cm}$

Therefore the smallest side is **5**.

Hence, the option (2) is the correct answer.

Section – B

6. Find the area of a triangular garden whose sides are $40\text{ m.}, 90\text{ m}$ **and** 70 m**. (use** $\sqrt{5} = 2.24$ **)**

Ans: Let $a = 40\text{ m}, \text{b} = 90\text{ m}$ and $\text{c} = 70\text{ m}$

The half perimeter,

$$s = \frac{(a+b+c)}{2}$$

$$\Rightarrow \frac{(40+90+70)}{2}$$

$$\Rightarrow \frac{200}{2}$$

$$s = 100$$

By Heron's formula of area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$$\Rightarrow \sqrt{100(100-40)(100-90)(100-70)}$$

$$\Rightarrow \sqrt{100 \times 60 \times 10 \times 30}$$

$$\Rightarrow 10\sqrt{18000}$$

$$\Rightarrow 10 \times 60 - \sqrt{5}$$

$$= 10 \times 134.4$$

$$= 1344\text{ m}^2.$$

The area of the triangular garden $= 1344\text{ m}^2$.

7. Find the cost of leveling a ground in the form of a triangle with sides $16m,\ 12m\ and\ 20m$ **at Rs.4 per sq.meter.**

Ans: Let the sides be $a = 16, b = 12, c = 20.$

By herons formula

$s = \frac{a+b+c}{2}$

$= \frac{16+12+20}{2}$

$= \frac{48}{2}$

$= 24$

The area of the triangle,

$\Rightarrow A = \sqrt{s(s-a)(s-b)(s-c)}$

$\Rightarrow A = \sqrt{(24-16)(24-12)(24-20)}$

$\Rightarrow A = \sqrt{24 \times 8 \times 12 \times 4}$

$\Rightarrow A = \sqrt{(2 \times 2 \times 3 \times 2)(2 \times 2 \times 2)(2 \times 3 \times 2)(2 \times 2)}$

$\Rightarrow A = 2 \times 2 \times 2 \times 2 \times 2 \times 3$

$\Rightarrow A = 96 \text{ m sq}$

Cost per meter $= 4$

Cost for $96 \text{ m} = 4 \times 96$

$= 384\text{hrs}$.

8. Find the area of a triangle, two sides of which are 8 cm **and** 11 cm **and the perimeter is** 32 cm.

Ans: Let **a, b, c** be the sides of the given triangle and **2s** be its perimeter such that $a = 8 \text{ cm}, b = 11 \text{ cm}$ and $2s = 32 \text{ cm}$

Now, $a + b + c = 2s$

$8 + 11 + c = 32$

$c = 13$

Therefore,

$s - a = 16 - 8 = 8,$

$s - b = 16 - 11 = 5,$

$s - c = 16 - 13 = 3$

Hence, the area of given triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{16 \times 8 \times 5 \times 3}$

$= 8\sqrt{30}\ \text{cm}^2.$

9. The area of an isosceles triangle is $12\ \text{cm}^2$. If one of its equal side is $5\ \text{cm}$. Find its base.

Ans: Let equal sides be $(a) = 5\text{cm}$ and base (b) =?

Area of an isosceles triangle $= 12\ \text{sq. cm}$

Area of an isosceles triangle

$= \frac{b}{4}\sqrt{4a^2 - b^2}$

$12 = \frac{b}{4}\sqrt{4 \times (5)^2 - b^2}$

$48 = b\sqrt{100 - b^2}$

Squaring both the sides, we get

$2304 = b^2\left(100 - b^2\right)$

$b^4 - 100b^2 + 2304 = 0$

$b^2 - 64b^2 - 36b^2 + 2304 = 0$

$b^2(b^2-64)-36(b^2-64)=0$

$(b^2-64)(b^2-36)=0$

either $b^2-64=0$

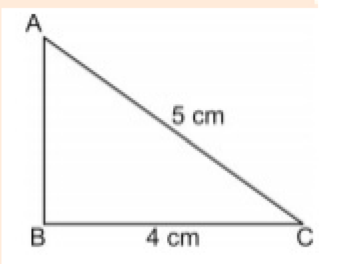
$\Rightarrow b^2=64 \Rightarrow b=\pm 8$

or $b^2-36=0$

$\Rightarrow b^2-36 \Rightarrow b=\pm 6$

Hence base $=8\text{cm}$, or 6cm.

11. Find the area of the adjoin figure if AB *and* BC



Ans: Since, $\angle B=90^\circ$ **ABC** is a right angle triangle.

Pythagoras Theorem,

$\Rightarrow AB^2+BC^2=AC^2$

$\Rightarrow AB^2+4^2=5^2$

$\Rightarrow AB^2=25-16$

$\Rightarrow AB^2=9$

$\Rightarrow AB=3\text{ cm}$

$\text{Area}(\Delta ABC) = \frac{1}{2}\times AB\times BC$

$=\frac{1}{2}\times 3\times 4$

$=6\text{cm}^2$

Section - C

12. The diagonals of a rhombus are 24 cm and 10 cm. Find its area and perimeter.

Ans: Find the area,

Area $=\frac{1}{2}\times 24\times 10$

$=120\text{ cm}^2$

Perimeter $s^2=\left(\frac{24}{2}\right)^2+\left(\frac{10}{2}\right)^2$

$=12^2+5^2$

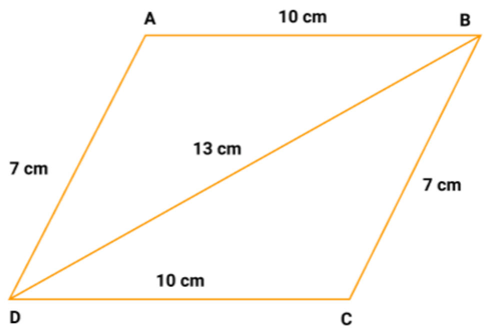
$=169$

$\text{s}=13$

The perimeter of the rhombus $=4\times 13=52\text{ cm}$.

13. Two side of a parallelogram are 10 cm and 7cm. One of its diagonals is 13 cm. Find the area.

Ans:



ABCD is parallelogram

$AB = CD = 10\text{ cm}$

$AD = CB = 7\text{ cm}$

Diagonal $BD = 13\text{ cm}$

Diagonal divides the parallelogram into two equal triangles

Find the area of triangle ABD

Area $= \sqrt{s(s-a)(s-b)(s-c)}$

$s = \dfrac{a+b+c}{2}$

$a = 10$

$b = 7$

$c = 13$

Substitute the values in the formula :

$s = \dfrac{10+7+13}{2}$

$s = \dfrac{10+7+13}{2}$

Area $= \sqrt{15(15-10)(15-7)(15-13)}$

Area $=34.6410161514$

Area of parallelogram $=2\times$ Area of triangle $=2\times 34.6410161514=69.2820\ \text{cm}^2$

Hence the area of parallelogram is $69.2820\ sq.cm$.

14. A rhombus shaped sheet with perimeter $40\ cm$ and one diagonal $12\ cm$], is painted on both sides at the rate of 5 per m^2. Find the cost of painting

Ans: Let ABCD be a rhombus, then $\text{AB}=\text{BC}=\text{CD}=\text{DA}=\text{x}$

Perimeter of rhombus $=40\ \text{cm}$

$\Rightarrow 4\text{x}=40\ \text{cm}\Rightarrow \text{x}=10\ \text{cm}$

$\therefore \text{AB}=\text{BC}=\text{CD}=\text{DA}=10\ \text{cm}$

In $\Delta\text{ABC},\text{S}=\dfrac{\text{a}+\text{b}+\text{c}}{2}=\dfrac{10+10+12}{2}=16\ \text{cm}$

ar $\Delta\text{ABC}=\sqrt{16(16-10)(16-10)(16-12)}=\sqrt{16\times 6\times 6\times 4}=48\ \text{cm}^2$

ar. $\text{ABCD}=2\times 48=96\ \text{cm}^2$

Cost of painting the sheet $=\text{Rs}(5\times 96\times 2)=\ Rs\ 960$

15. The sides of a quadrilateral ABCD are $6\ \text{cm}, 8\ \text{cm}, 12\ \text{cm}$ and $14\ \text{cm}$ (taken in order) respectively, and the angle between the first two sides is a right angle. Find its area.

Ans: Applying Pythagoras theorem in ΔABC, we get

$\text{AC}=\sqrt{\text{AB}^2+\text{BC}^2}=\sqrt{6^2+8^2}=\sqrt{36+64}=\sqrt{100}=10\ \text{cm}$

So, the area of $\Delta\text{ABC}=\dfrac{1}{2}\times$ base $\times$ height

$=\dfrac{1}{2}\times\text{AB}\times\text{BC}$

$= \frac{1}{2} \times 6 \times 8 = 24$

Now, in ΔACD, we have,

$AC = 10$ cm,

$CD = 12$ cm,

$AD = 14$ cm

Now, in ΔACD, we have $AC = 10$ cm, $CD = 12$ cm, $AD = 14$ cm

According to Heron's formula the area of triangle $(A) = \sqrt{[s(s-a)(s-b)(s-c)]}$

where, $2s = (a+b+c)$

Here, $a = 10$ cm, $b = 12$ cm, $c = 14$ cm

$s = \frac{(10+12+14)}{2} = \frac{36}{2} = 18$

Area of $\Delta ACD = \sqrt{[18 \times (18-10)(18-12)(18-14)]}$

$= \sqrt{(18 \times 8 \times 6 \times 4)}$

$= \sqrt{(2 \times 3 \times 3 \times 2 \times 2 \times 2 \times 2 \times 3 \times 2 \times 2)}$

$= \sqrt{[(2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3) \times 2 \times 3]}$

$= 2 \times 2 \times 2 \times 3 \times \sqrt{6}$

$= 24\sqrt{6}$

So, total area of quadrilateral $ABCD = \Delta ABC + \Delta ACD$

$= 24 + 24\sqrt{6}$

$= 24(\sqrt{6} + 1)$.

16. The perimeter of an isosceles triangle is 32 cm. The ratio if the equal side to its base is 32. Find the area of the triangle.

Ans: The ratio of the equal side to the base is **32.**

Let the sides be $3\text{x}, 2\text{x}$. Let the third $=3\text{x}$

Given, perimeter $=32$

We know that the perimeter is equal to the sum of the sides. Thus, $\Rightarrow 3x+2x+3x=32$

$\Rightarrow 8x=32$

$\Rightarrow x=4$

$\Rightarrow \frac{32}{2}=16$

Thus, the sides are $12\text{ cm}, 8\text{ cm}, 12\text{ cm}$

Thus, Area of the triangle $=\sqrt{\frac{32}{2}(16-12)(16-8)(16-12)}$

$=\sqrt{16\times 4\times 8\times 4}$

$=32\sqrt{2}\text{ cm}^2$.

17. The sides of a triangular field are $41\text{ m}, 40\text{ m}$ and 9 m. Find the number of flower beds that can be prepared in the field, if each flower bed needs 900 cm^2 space.

Ans: By Heron's formula. Area of a triangular= $\cdot\sqrt{s}\times(s-a)(s-b)(s-c)$, where **a, b, c** are sides of the triangle and s is the semi perimeter. so, area of the field $=\sqrt{[45\times(45-41)(45-40)(45-9)]}$

$=\sqrt{(45\times 4\times 5\times 36)}$

$=\sqrt{32400}$

$= 180\ m^2$

$= 1800000\ cm^2$

now, space needed for a flower bed $= 900\ cm^2$

so, number of flower beds $= \frac{1800000}{900}$

$= 2000.$

18. The perimeter of a triangular ground is $420\ m$ and its sides are in the ratio 6:7:8. Find the area of the triangular ground.

Ans: The perimeter of triangular field $= 420\ m$.

Given that the ratios of the sides are **6:7:8**

Sum of the ratios $= 6+7+8 = 21$

Length of first side of the field

$= \frac{6}{21} \times 420$

$= 6 \times 20$

$= 120\ m$

Length of second side of the field $= \frac{7}{21} \times 420 = 7 \times 20 = 140\ m$

Length of third side of the field $= \frac{8}{21} \times 420 = 8 \times 20 = 160\ m$

According to Heron's formula the area (A) of triangle with sides a, b and c is given as,

$A = \sqrt[2]{s(s-a)(s-b)(s-c)}]$ where $2s = (a+b+c)$

Here $a = 120\ m, b = 140\ m, c = 160\ m,$

$$s = \frac{(120+140+160)}{2} = \frac{420}{2} = 210$$

Area of triangular field $= \sqrt{210 \times (210-120)(210-140)(210-160)}$

$= \sqrt{(210 \times 90 \times 70 \times 50)}$

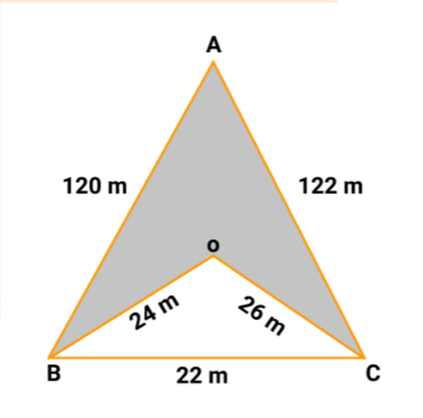
$= \sqrt{(3 \times 7 \times 3 \times 3 \times 7 \times 5 \times 10000)}$

$= \sqrt{[(7 \times 3 \times 100 \times 7 \times 3 \times 100) \times 3 \times 5]}$

$= 2100\sqrt{15}$.

Section - D

19. Calculate the area of the shaded region.



Ans: Area of shaded region $= \text{ar}\,\Delta\text{ABC} - \text{ar}\,\Delta\text{DBC}$

$\text{ar}\,\Delta\text{ABC} = \sqrt{s(s-a)(s-b)(s-c)}$

$s = \frac{a+b+c}{2} = \frac{122+22+120}{2} = 132\text{ m}$

$\therefore \text{ar}\,\Delta\text{ABC} = \sqrt{132(132-122)(132-22)(132-120)}$

$= \sqrt{132 \times 10 \times 110 \times 12}$

$= \sqrt{11 \times 12 \times 10 \times 10 \times 11 \times 12}$

$=10\times 11\times 12=1320\text{ m}^2$

$ar\Delta BDC=\sqrt{s(s-a)(s--b)(s-c)}$

$s=\frac{a+b+c}{2}=\frac{36+22+24}{2}=36\text{ m}$

$\therefore \text{ar}\,\Delta\text{BDC}=\sqrt{36(36-26)(36-22)(36-24)}$

$=\sqrt{36\times 10\times 14\times 12}$

$=\sqrt{12\times 3\times 2\times 5\times 2\times 7\times 12}$

$=2\times 12\sqrt{105}=24\times 10.24=245.76\text{ m}^2$

$\therefore$ Area of shaded region $=1320-245.76=1074.24\text{ m}^2\approx 1074\text{ m}^2$

20. If each sides of a triangle is double, then find the ratio of area of the new triangle thus formed and the given triangle.

Ans: Let a,b and c denotes the length of the sides of the triangle.

Area of the triangle, $A_1=\sqrt{s(s-a)(s-b)(s-c)}$, where s is the semi-perimeter of the triangle. So, semi perimeter $s=\frac{a+b+c}{2}$

When the sides of the triangle are doubled, we get $s'=\frac{2a+2b+2c}{2}=a+b+c=2s$, where s' is the semi-perimeter of the new triangle

Area of the new triangle, $A_2=\sqrt{s'\left(s'-2a\right)\left(s'-2b\right)\left(s'-2c\right)}$

$=\sqrt{2s(2s-2a)(2s-2b)(2s-2c)}$

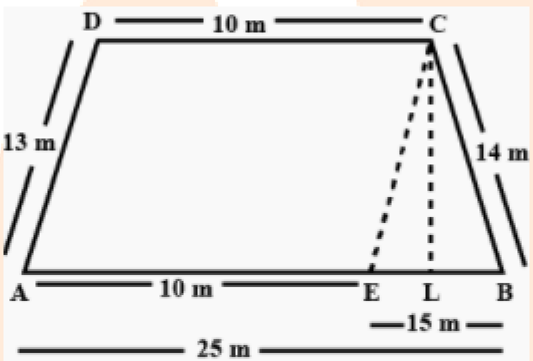
$=\sqrt{16s(s-a)(s-b)(s-c)}$

$=4\sqrt{s(s-a)(s-b)(s-c)}=4A_1$

Therefore, the ratio of the area of new triangle to the given triangle $= \frac{A_2}{A_1} = \frac{4\,A_1}{A_1} = 4:1$

21. A field is in the shape of a trapezium whose parallel sides are 25 m and 10 m. If its non-parallel sides are 14 m and 13 m, find its area.

Ans:



Let ABCD be a trapezium with,

$\text{AB} = 25\text{ m}$

$\text{CD} = 10\text{ m}$

$\text{BC} = 14\text{ m}$

$\text{AD} = 13\text{ m}$

Draw CE || DA. So, ADCE is a parallelogram with, $\text{CD} = \text{AE} = 10\text{ m}$

$\text{CE} = \text{AD} = 13\text{ m}$

$\text{BE} = \text{AB} - \text{AE} = 25 - 10 = 15\text{ m}$

In ΔBCE, the semi perimeter will be, $s = \frac{a+b+c}{2}$

$\text{s} = \frac{14+13+15}{2}$

$\text{s} = 21\text{ m}$

Area of ΔBCE,

$$A=\sqrt{s(s-a)(s-b)(s-c)}$$

$$=\sqrt{21(21-14)(21-13)(21-15)}$$

$$=\sqrt{21(7)(8)(6)}$$

$$=\sqrt{7056}$$

$$=84\ m^2$$

Also, area of ΔBCE is, $A=\frac{1}{2}\times$ base $\times$ height

$$84=\frac{1}{2}\times 15\times CL$$

$$\frac{84\times 2}{15}=CL$$

$$CL=\frac{56}{5}\ m$$

The area of trapezium is, $A=\frac{1}{2}$ (sum of parallel sides) (height)

$$A=\frac{1}{2}\times(25+10)\left(\frac{56}{5}\right)$$

$$A=196\ m^2$$

Therefore, the area of the trapezium is $196\ m^2$.

22. An umbrella is made by stitching 10 triangular pieces of cloth of 5 different colour each piece measuring $20\ cm, 50\ cm$ and $50\ cm$. How much cloth of each colour is required for one umbrella? $(\sqrt{6}=2.45)$

Ans: Area of a triangle $=\frac{1}{2}$ bh

Hear, $b=20$

$h=\sqrt{50^2-10^2}=\sqrt{2500-100}$

$=\sqrt{2400}$

$=\sqrt{6\times400}\cdot20\sqrt{6}$

$\therefore$ Area $=\frac{1}{2^2}\times20.20\sqrt{6}$

$=10\times20\sqrt{6}=200\sqrt{6}$

$=200\times245=490\text{ cm}^2$.

Each colour. cloth is used 2 times.

$\therefore$ The area of each colour cloth required for one umbrella $=490\times2$

$=980\text{cm}^2$

23. A triangle and a parallelogram have the same base and some area. If the sides of the triangle are $26\text{ cm}, 28\text{ cm}$ and 30 cm and the parallelogram stands on the base 28 cm, find the height of the parallelogram.

Ans: Find Perimeter of Triangle,

$2\text{ S}=26+28+30=84$

$\Rightarrow\text{S}=42\text{ cm}$

Use Heron's formula,

Area $\sqrt{s(s-a)(s-b)(s-c)}$

Area $=\sqrt{42(42-26)(42-28)(42-30)}$

$=\sqrt{42\times16\times14\times12}$

Area $=336\text{ cm}^2$

Area of parallelogram = Area of triangle

$\Rightarrow h \times 28 = 336$

$\Rightarrow h = 12\text{ cm}$

Height of parallelogram $= 12\text{ cm}$.

1 Mark Questions

1. The measure of each side of an equilateral triangle whose area is $\sqrt{3}\text{ cm}^2$ is,

(A) 8 cm

(B) 2 cm

(C) 4 cm

(D) 16 cm

Ans: Correct answer option (B) 2 cm

2. Measure of each side of an equilateral triangle is 12 cm. Its area is given by

(A) $9\sqrt{3}$ sq cm

(B) $18\sqrt{3}$ sq cm

(C) $27\sqrt{3}$ sq cm

(D) $36\sqrt{3}$ sq cm

Ans: Correct answer option (D) $36\sqrt{3}$ sq cm

3. Two adjacent side of a parallelogram are 74cm and 40cm one of its diagonals is 102cm. Area of the ||gram is

(A) 612sqm

(B) 1224sqm

(C) 2448sqm

(D) 4896sqm

Ans: Correct answer option (C) 2448sqm

4. In heron's formula $\sqrt{s*(s-a)*(s-b)*(s-c)}$, what is the value of s if a, b and c are sides of the triangle?

A) $\frac{a+b+c}{4}$

B) $a+b+c$

C) $\frac{a+b+c}{2}$

D) $2a+2b+2c$

Ans: C

5. The perimeter of a triangle is $60\,\text{cm}$. If its sides are in the ratio $1:\ 3:\ 2,$ then its smallest side is

(A) 15

(B) 5

(C) 10

(d) none of these.

Ans: Correct answer option (C) **10**

6. The perimeter of a triangle is $36\,\text{cm}$. If its sides are in the ratio $1:\ 3:\ 2$, then its largest side is

(a) 6

(b) 12

(c) 18

(d) none of these.

Ans: Correct answer option (c) **18**

7. If the perimeter of a rhombus is $20\,\text{cm}$ and one of the diagonals is $8\,\text{cm}$. The area of the rhombus is

(a) 24sqcm

(b) 48sqcm

(c) 50sqcm

(d) 30sqcm

Ans: Correct answer option (a) 24sqcm

8. One of the diagonals of a rhombus is $12\,\text{cm}$ and area is 54sqcm. the perimeter of the rhombus is

(a) $72\,\text{cm}$

(b) $\sqrt[3]{10}\ \text{cm}$

(c) $\sqrt[6]{10}\ \text{cm}$

(d) $\sqrt[12]{10}\ \text{cm}$

Ans: Correct answer option (d) $\sqrt[12]{10}\ \text{cm}$

9. The side of a triangle is $12\text{ cm}, 16\text{ cm}$**, and** 20 cm**. Its area is**

(A) 100cm^2

(B) 90cm^2

(C) 96cm^2

(D) 120cm^2**.**

Ans: Correct answer option (C) 96cm^2

10. The side of an equilateral triangle is $4\sqrt{3}\text{ cm}$**. Its area is.**

(A) $12\sqrt{3}\text{cm}^2$

(B) $12\sqrt{6}\text{cm}^2$

(C) $12\sqrt{10}\text{cm}^2$

(D) $6\sqrt{10}\text{cm}^2$**.**

Ans: (A) $12\sqrt{3}\text{cm}^2$

11. It the perimeter of a rhombus is 20 sq cm **and one of the diagonals is** 8 cm**. Then the area of the rhombus is**

(A) 40sqcm

(B) 24sqcm

(C) 20sqcm

(D) 13sqcm**.**

Ans: Correct answer is option (B) 24sqcm

12. One of the diagonals of a rhombus is 12 cm and Its area is 54sqcm. The perimeter of

the rhombus is.

(A) 10 cm

(B) 8 cm

(C) 6 cm

(D) $12\sqrt{10}\text{ cm}$**.**

Ans: (D) $12\sqrt{10}\text{ cm}$.

13. The lengths of the side of a triangular park are $90\text{ m}, 70\text{ m}$ and 40 m, find Its area.

(A) 1340sqm

(B) 134sqm

(C) 140sqm

(D) 1444sqm

Ans: (B) 1344sqm

14. An equilateral triangle has a side 50 cm long. Find the area of the triangles.

(A) $625\sqrt{3}\text{m}^2$

(B) $625\sqrt{6}\text{m}^2$

(C) $256\sqrt{6}\text{m}^2$

(D) $625\sqrt{10}\,m^2$

Ans: (A) $625\sqrt{3}\,m^2$

15. The area of an isosceles triangle is $12cm^2$. If one of the equal side is $5\,cm$, then the length of the base is

(A) $4\,cm$

(B) $5\,cm$

(C) $6\,cm$

(D) $8\,cm$

Ans: (C) $6\,cm$

16. Find the area of triangle whose side is $6\,cm, 10\,cm$ and $15\,cm$.

(A) $404.9 sqcm$

(B) $405.9 sqcm$

(C) $402.9 sqcm$

(D) $410 sqcm$

Ans: (A) $404.9 sqcm$

17. If side of equilateral triangle is $25\,m$. Its area is

(a) $\frac{625}{4}\sqrt{3} sqcm$

(b) $54\sqrt{3} sqcm$

(c) $5\sqrt{3}\text{sqcm}$

(d) $\sqrt{3}\text{sqcm}$

Ans: (a) $\frac{625}{4}\sqrt{3}\text{sqcm}$

18. The perimeter of an equilateral triangle is 48 cm. Its area is

(a) $18\sqrt{3}\text{sqcm}$

(b) $72\sqrt{3}\text{sqcm}$

(c) $64\sqrt{3}\text{sqcm}$

(d) $60\sqrt{3}\text{sqcm}$

Ans: (c) $64\sqrt{3}\text{sqcm}$

19. If area of isosceles triangle is 48cm^2 and length of one of its equal sides is 10 m, then what is the base?

(a) 16 cm **or** 12 cm

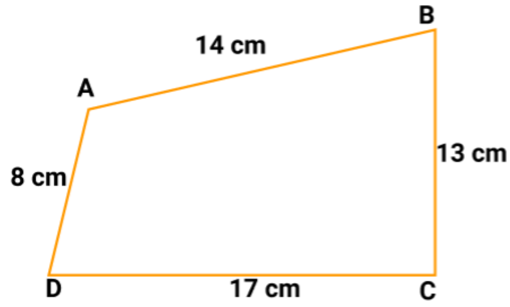
(b) 12 cm **or** 14 cm

(c) 14 cm **or** 16 cm

(d) 16 cm **or** 18 cm

Ans: (a) 16 cm or 12 cm

20. If $AB = 14\text{ cm}, BC = 13\text{ cm}, CD = 17\text{ cm}, DA = 8\text{ cm}$ and $AC = 15\text{ cm}$ then area of quadrilateral ABCD is



(a) 150sq cm

(b) 144sq cm

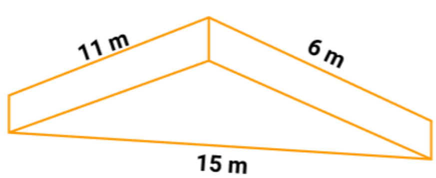
(c) 142sq cm

(d) 100sqcm

Ans: (b) 144 sq cm

2 Marks Questions

1. There is slide in a park. One of its side walls has been painted in some colour with a message "KEEP THE PARK GREEN AND CLEAN", (see figure). If the sides of the wall are $15\text{ m}, 11\text{ m}$ and 6 m, find the area painted in colour.



Ans: Sides of coloured triangular wall are $15\text{ m}, 11\text{ m}$ and 6 m.

$\therefore$ Semi-perimeter of coloured triangular wall

$$= \frac{15+11+6}{2} = \frac{32}{2} = 16\text{ m}$$

Now, Using Heron's formula,

Area of coloured triangular wall

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{16(16-15)(16-11)(16-6)}$

$= \sqrt{16 \times 1 \times 5 \times 10} = 20\sqrt{2} m^2$

Therefore, the painted in blue colour $= 20\sqrt{2} m^2$

2. Find the area of a triangle two sides of which are 18 cm and 10 cm and the perimeter is 42 cm.

Ans: Given: $a = 18\text{ cm}, b = 10\text{ cm}$

Since Perimeter $= 42\text{ cm}$

$\Rightarrow a + b + c = 42$

$\Rightarrow 18 + 10 + c = 42$

$\Rightarrow c = 42 - 28 = 14\text{ cm}$

Therefore, Semi-perimeter of triangle

$= \frac{18 + 10 + 14}{2} = 21\text{ cm}$

Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{21(21-18)(21-10)(21-14)}$

$= \sqrt{21 \times 3 \times 11 \times 7}$

$= \sqrt{7 \times 3 \times 3 \times 11 \times 7}$

$= 21\sqrt{11}$

$= 21 \times 3.3$

$= 69.3\text{ cm}^2$.

3. Sides of a triangle are in the ratio of 12:17:25 and its perimeter is 540 cm. Find its area.

Ans: Let the sides of the triangle be 12x, 17x and 25x

Therefore, $12x+17x+15x=540$

$\Rightarrow 54x=540 \Rightarrow x=10$

- The sides are $120\text{ cm},170\text{ cm}$ and 250 cm.

Semi-perimeter of triangle $(s)=\dfrac{120+170+250}{2}=270\text{ cm}$

Now, Area of triangle $=\sqrt{s(s-a)(s-b)(s-c)}$

$\sqrt{270(270-120)(270-170)(270-250)}$

$=\sqrt{270\times 150\times 100\times 20}$

$=9000\text{ cm}^2$

4. An isosceles triangle has perimeter 30 cm and each of the equal sides is 12 cm. Find the area of the triangle.

Ans: Given: $a=12\text{ cm}, b=12\text{ cm}$

Since Perimeter $=30\text{ cm} \Rightarrow a+b+c=30$

$\Rightarrow 12+12+c=30$

$\Rightarrow c=30-24=6\text{ cm}$

Semi-perimeter of triangle $=\dfrac{12+12+6}{2}=15\text{ cm}$

$\therefore$ Area of triangle $=\sqrt{s(s-a)(s-b)(s-c)}$

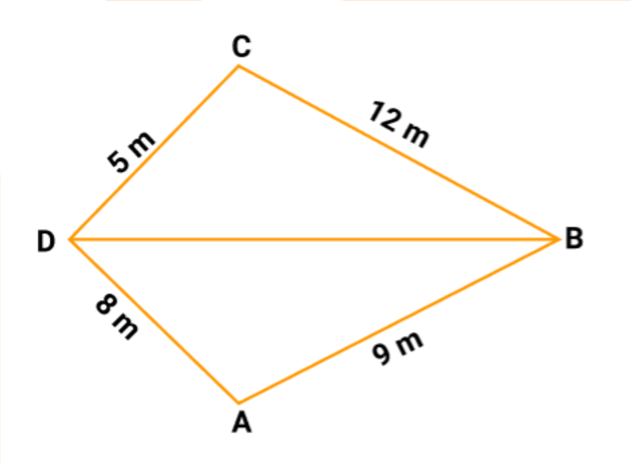
$\sqrt{15(15-12)(15-12)(15-6)}$

$= \sqrt{15 \times 3 \times 3 \times 9}$

$= \sqrt{5 \times 3 \times 3 \times 3 \times 3 \times 3}$

$= 9\sqrt{15}\ \text{cm}^2$

5. A park, in the shape of a quadrilateral ABCD **has** $\angle C = 90^\circ, AB = 9\text{ m}, BC = 12\text{ m}, CD = 5\text{ m}$ **and** $AD = 8\text{ m}$**. How much area does it occupy?**



Ans: Since **BD** divides quadrilateral **ABCD** in two triangles:

(i) Right triangle BCD and (ii) ΔABD.

In right triangle BCD, right angled at C,

Therefore, Base $= CD = 5\text{ m}$ and Altitude $= BC = 12\text{ m}$

Area of $\Delta BCD = \frac{1}{2} \times CD \times BC$

$= \frac{1}{2} \times 5 \times 12 = 30\text{ m}^2$

In $\Delta ABD, AB = 9\text{ m}, AD = 8\text{ m}$

And $BD = \sqrt{CD^2 + BC^2}$ [Using Pythagoras theorem]

$\Rightarrow BD = \sqrt{(5)^2 + (12)^2}$

$= \sqrt{25+144} = \sqrt{169} = 13\text{ m}$

Semi=perimeter of $\Delta ABD = \dfrac{9+8+13}{2} = 15\text{ m}$

Using Heron's formula,

Area of $\Delta ABD = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{15(15-9)(15-8)(15-13)}$

$= \sqrt{15 \times 6 \times 7 \times 2}$

$= \ = 6\sqrt{35} = 6 \times 5.91\text{ m}^2$

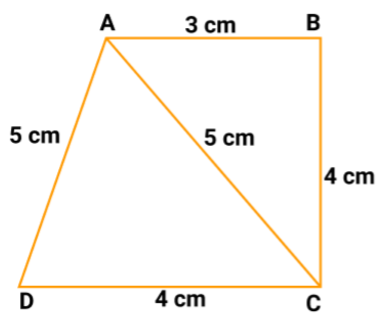
$= 35.4\text{ m}^2$ (approx.)

Area of quadrilateral $ABCD =$ Area of $\Delta BCD +$ Area of ΔABD

$= 30 + 35.4$

$= 65.4\text{ m}^2$

6. Find the area of a quadrilateral $ABCD$ in which $AB = 3\text{ cm}, BC = 4\text{ cm}, CD = 4\text{ cm}, DA = s\text{ cm}$ and $AC = 5\text{ cm}$.



Ans: In quadrilateral **ABCE**, diagonal **AC** divides it in two triangles, ΔABC and ΔADC.

In ΔABC, Semi-perimeter of $\Delta ABC = \frac{3+4+5}{2} = 6 \text{ cm}$

Using Heron's formula,

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{6(6-3)(6-4)(6-5)}$

$= \sqrt{6 \times 3 \times 2 \times 1} = 6 \text{ cm}^2$

Again, In ΔADC, Semi-perimeter of $\Delta ADC = \frac{4+5+5}{2} = 7 \text{ cm}$

Using Heron's formula, Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{7(7-4)(7-5)(7-5)}$

$= \sqrt{7 \times 3 \times 2 \times 2} = 2\sqrt{21}$

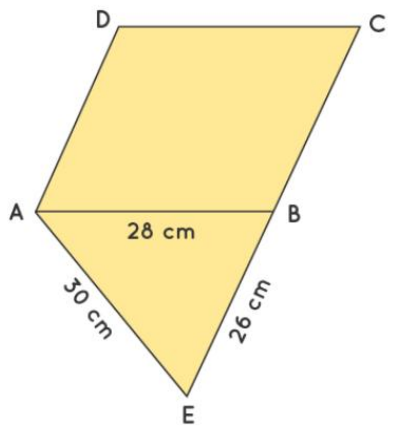
$= 2 \times 4.6 = 9.2 \text{ cm}^2$ (approx.)

Area of quadrilateral $ABCD =$ Area of $\Delta ABC +$ Area of ΔADC

$= 6 + 9.2$

$= 15.2 \text{ cm}^2$

7. A triangle and a parallelogram have the same base and the same area. If the sides of the triangle are $26 \text{ cm}, 29 \text{ cm}$ and 30 cm and the parallelogram stands on the base 28 cm, find the height of the parallelogram.



Ans: For $\triangle ABE, a = 30\,cm, b = 26\,cm, c = 28\,cm$

Semi Perimeter: $(s) =$ Perimeter $/2$

$s = (a+b+c)/2$

$= (30+26+28)/2$

$= 42\,cm$

By using Heron's formula,

Area of a $\triangle ABE = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{42(42-30)(42-28)(42-26)}$

$= \sqrt{42 \times 12 \times 14 \times 16} \; = 336\,cm^2$

Area of parallelogram $ABCD =$ Area of $\triangle ABE$ (given)

Base $\times$ Height $= 336\,cm^2$

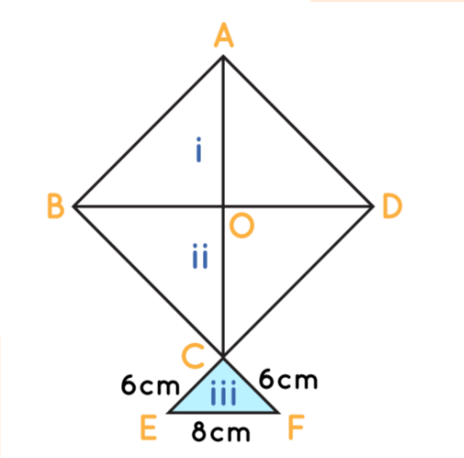
$28\,cm \times$ Height $= 336\,cm^2$

On rearranging, we get

Height $= 336/28\,cm = 12\,cm$

Thus, height of the parallelogram is $12\,cm$.

8. A kite is in the shape of a square with a diagonal 32 cm and an isosceles triangle of base 8 cm and sides 6 cm each is to be made of three different shades as shown in figure.



How much paper of each side has been used in it?

Ans: Heron's formula for the area of a triangle $=\sqrt{s(s-a)(s-b)(s-c)}$

Where a, b, and c are the sides of the triangle, and

$s=$ Semi-perimeter = Half the Perimeter of the triangle $=(a+b+c)/2$

Given diagonal $BD=AC=32\text{cm}$, then $OA=1/2AC=16\text{cm}$.

So square ABCD is divided into two isosceles triangles ABD and CBD of base 32cm and height 16cm.

Area of $\triangle\text{ABD}=1/2\times$ base $\times$ height $=(32\times16)/2 \ =256\text{cm}^2$

Since the diagonal divides the square into two equal triangles. Therefore,

Area of $\triangle\text{ABD}=$ Area of $\Delta\text{CBD}=256\text{cm}^2$

Now, for ΔCEF

Semi Perimeter(s) $=(a+b+c)/2$

$s=(6+6+8)/2$

$s = 20/2$

$s = 10\,\text{cm}$

By using Heron's formula,

Area of $\Delta CEF = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{10(10-6)(10-6)(10-8)}$

$= \sqrt{10 \times 4 \times 4 \times 2}$

$= 8\sqrt{5}$

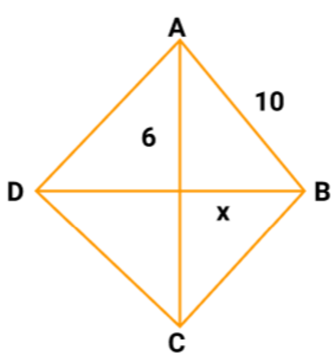
$= 8 \times 2.24$

$= 17.92\,\text{cm}^2$

Thus, the area of the paper used to make region $\text{I} = 256\,\text{cm}^2$, region $\text{II} = 256\,\text{cm}^2$, and region $III = 17.92\,\text{cm}^2$

10. The perimeter of a rhombus ABCD is 40 cm. find the area of rhombus of Its diagonals BD measures 12 cm

Ans: Side $= \dfrac{\text{perimeter}}{4} = 10\,\text{cm}$



$10^2 - 6^2 = x^2$

$x^2 = 64 \Rightarrow x = 8$

$\therefore$ other diagonal $= 2x = 16\text{cm}$

$\therefore$ Area $= \frac{1}{2} d_1 d_2$

$= \frac{1}{2} \times 16 \times 12 = 16 \times 6 = 96\text{cm}^2$

11. Find area of triangle with two sides as 18 cm and 10 cm **and the perimeter is** 42 cm**.**

Ans: Let $a = 18\text{ cm}, \text{b} = 10\text{ cm}$

Perimeter $= 42\text{ cm}$

$a + b + c = 42\text{ cm}$

So, $\text{C} = 14\text{ cm}$

$S = \frac{a+b+c}{2} = \frac{18+10+14}{2} = 21\text{ cm}$

new area of triangles $= \sqrt{21(21-18)(21-10)(21-14)}$

$= \sqrt{21 \times 3 \times 11 \times 7}$

$= 21\sqrt{11}\text{sqcm}$

12. Find the area of in isosceles triangle, the measure of one of Its equals side being $\cdot b$ **and the third side '** a **'.**

Ans: Here

$S = \frac{a+b+c}{2}$ units $= \frac{a+2b}{2}$ units

$\therefore$ area of $\Delta = \sqrt{\left(\frac{a+2b}{2}\right)\left(\frac{a+2b}{2} - a\right)\left(\frac{a+2b}{2} - b\right)\left(\frac{a+2b}{2} - c\right)}$

$= \sqrt{\left(\frac{a+2b}{2}\right)\left(\frac{2b-a}{2}\right)\frac{a}{2} \times \frac{a}{2}} squnits$

$= \frac{a}{4}\sqrt{4b^2 - a^2} sq$ units

13. Find the cost of leveling the ground in the form of a triangle having its sides are $40\text{m}, 70\text{ m}$ and 90 m at $Rs\ 8$ per square meter. [use $\sqrt{5} = 2.24$]

Ans: Here $S = \frac{40+70+90}{2}\text{ m} = 100\text{ m}$

- Area of a triangular ground $= \sqrt{100(100-40)(100-70)(100-90)} sqm$

$= \sqrt{100 \times 60 \times 30 \times 10}\text{sqm}$

$= (10 \times 10 \times \sqrt[6]{5})\text{sqm}$

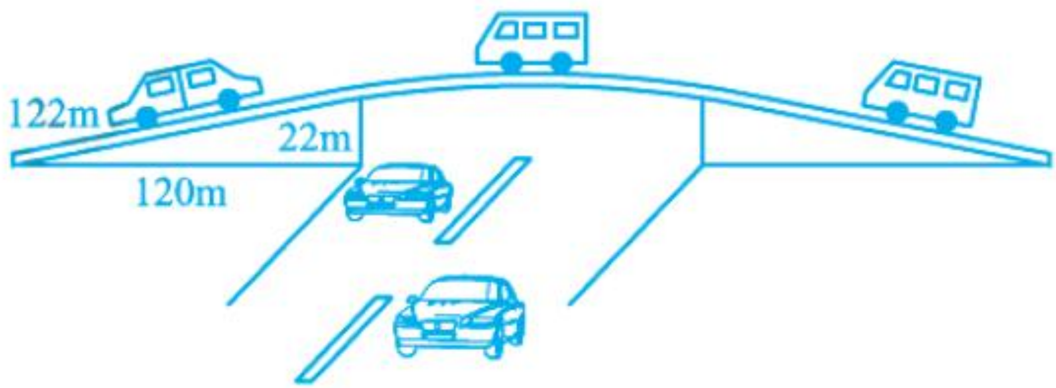
$= (600 \times 2.24)\text{sqm}$

$= 1344 sqm$

- Cost of leveling the ground $= \text{Rs}(8 \times 1344)$

$= \text{Rs}10752$

14. The triangular side's walls of a flyover have been used for advertisements. The sides of the walls are $122\text{ m}, 22\text{ m}$ and 120 m. The advertisement yield on earning of $Rs\ 5000$ per m^2 per year. A company hired one of its walls for 4 months. How much rent did it pay?



Ans: The lengths of the sides of the walls are $122\text{ m}, 22\text{ m}$ and 120 m.

As,

$120^2 + 22^2$

$= 14400 + 484$

$= 14884$

$= (122)^2$

$\therefore$ Walls are in the form of right triangles

Area of one wall $= \frac{1}{2} \times \text{Base} \times \text{height}$

$= \frac{1}{2} \times 120 \times 22\,\text{sq m}$

$= 1320\,\text{sq m}$

Rent $= Rs5000/\text{sq m}$ per year

$\therefore$ Rent for 4 month $= \text{Rs}\left[\frac{5000 \times 1320 \times 4}{12}\right] = Rs22,00,000$

.

15. Find the perimeter and area of a triangle whose sides are of length $2\text{ cm}, 5\text{ cm}$ **and** $5\text{cm}.$

Ans: Here, $a=2$ cm, $b=5$ cm and $c=5$ cm

$\therefore$ Perimeter $=a+b+c=(2+5+5)=12$ cm

$S=$ semi perimeter

$=\frac{12}{2}=6$ cm

Using Heron's formula,

$\therefore$ Area of triangle $=\sqrt{s(s-a)(s-b)(s-c)}sqcm$

$=\sqrt{6(6-2)(6-5)(6-5)}\text{sqcm}$

$=\sqrt{24}\text{sqcm}$

$=4.9\text{sqcm}$

16. There is a slide in a park. One of its sides wall has been painted in some colour with a message 'KEEP THE CITY GREEN AND CLEAN".

If the sides of the wall are 15 m, 11 m and 6 m. Find the area painted in colour.

Ans: The sides of the wall is in the triangular from with sides,

$A=15$ m, $b=6$ m and $c=11$ m

$\therefore s=\frac{15+6+11}{2}$ m

$=16$ m

- Area to be painted in colour = Areas of the side wall

$\sqrt{s(t-a)(s-b)(s-c)}\,\text{sq}\,cm$

$\sqrt{16(16-5)(16-6)(16-11)}sqm$

$=\sqrt[4]{50}\text{sqm}$

$= \sqrt[2]{2} sqm$

17. Find the area of isosceles triangle whose side is $14\text{ m}, 12\text{ m}, 14\text{ m}$ **?**

Ans: Find the semi perimeter,

$$S = \frac{14+12+14}{2} = 20\text{mt}$$

Ares of isosceles triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

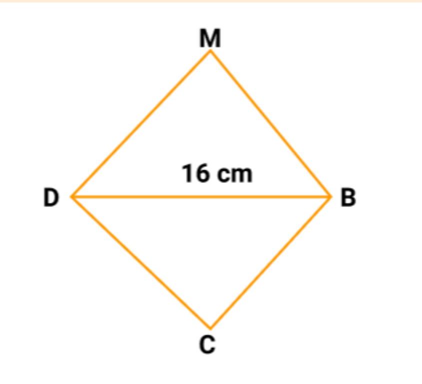
$$= \sqrt{20(20-14)(20-12)(20-14)}$$

$$= \sqrt{20 \times 6 \times 8 \times 6}$$

$$= 6\sqrt{160} = 6 \times 12.6$$

$$= 75.6$$

18. The perimeter af a rhombus MBCD **is** 60 cm**. find the area of the rhombus of Its diagonal BD measures** 16 cm?



Ans: As side of rhombus are equal.

$$\therefore AB = BC = CD = DA = \frac{60}{4} = 15\text{ cm } in \Delta \text{ABD}$$

$S=\frac{15+15+16}{2}=23\text{ cm}$

So,

Area of, $\Delta ABD=\sqrt{23(23-15)(23-15)(23-16)}$

$=\sqrt{23\times 8\times 8\times 7}=8\sqrt{23\times 7}$

$=8\times 12.7$

$=101.6\text{sqcm}$

Area of rhombus $=2\times 101.6$

$=203.2\text{sqcm}$

19. Find the cost of leveling the ground in the from of a triangle having Its side as 70 cm, 50 cm, and 60 cm, at $Rs\ 7$ per square meter.

Ans: Find the perimeter

$S=\frac{70+50+60}{2}$

$=\frac{180}{2}$

$=90\text{ cm}$

$\therefore$ area of triangle $=\sqrt{90(90-70)(90-50)(90-60)}$

$=\sqrt{90\times 20\times 40\times 30}$

$=1469.7\text{sqm}$

Cost of leveling the ground $=\text{RS}(7\times 1469.7)$

$=10287.9$

20. Find the area of a triangle two side of the triangle are 18 cm, and 12 cm. and the perimeter is 40 cm.

Ans: Let $a = 18\text{ cm}, b = 12\text{ cm}$ and $C = ?$

$$\text{So}, a + b + c = 40\text{ cm}$$

$$18 + 12 + C = 40$$

$$C = (40 - 30)\text{cn} = 10\text{ cm}$$

$$S = \frac{18 + 12 + 10}{2} = 20\text{ cm}$$

Therefore, the area of triangle $= \sqrt{20(20-18)(20-12)(20-10)}$

$$= \sqrt{20 \times 2 \times 8 \times 10}\text{sqcm}$$

$$= 56.56\text{sqcm}$$

21. Find the area of triangle whose side is $42\text{ m}, 56\text{ m}$ and 70 m ?

Ans: Find the semi perimeter

$$S = \frac{42 + 56 + 70}{2}\text{ m} = \frac{168}{2}\text{ m} \text{ or } \mathbf{84}$$

$\therefore$ Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{84(84-42)(84-56)(84-70)}\text{sqm}$$

$$= 42 \times 28\text{sqcm}$$

$$= 1176\text{sqcm}$$

22. Find the area of an isosceles triangle, the measure of one of Its equal side being b and the third side a.

Ans: Find the perimeter,

$$S=\frac{a+b+b}{2}\text{ units}$$

$$=\frac{a+2b}{2}\text{ units}$$

Area of triangle $=\sqrt{\frac{a+2b}{2}\times\left(\frac{a+2b}{2}-a\right)\left(\frac{a+2b}{2}-a\right)\left(\frac{a+2b}{2}-a\right)}\text{units}$

$$=\sqrt{\left(\frac{a+2b}{2}\right)\times\left(\frac{2b-a}{2}\right)\times\frac{a}{2}\times\frac{a}{2}}$$

$$=\frac{a}{4}\sqrt{4b^2-a^2}\text{ squnits}$$

23. Find the area of equilateral triangle whose side is 12 cm using Heron's formula.

Ans: Find the area of equilateral triangle,

$$S=\frac{12+12+12}{2}\text{cm}$$

$$=\frac{36}{2}\text{cm}=18\text{ cm}$$

$\therefore$ Area of equilateral $=\sqrt{s(s-a)(s-b)(s-c)}$

$$=\sqrt{18(18-12)(18-12)(18-12)}$$

$$=\sqrt{18\times6\times6\times6}$$

$$=36\sqrt{3}\text{sqcm}$$

24. Find the area of isosceles triangle whose equal side is $6\text{ cm},6\text{ cm}$ and 8 cm.

Ans: Find the area of isosceles triangle,

$S = \frac{6+6+8}{2}$ cm

$= \frac{20}{2} = 10$ cm

$\therefore$ Area of isosceles triangle $= \sqrt{10(10-6)(10-6)(10-8)}$

$= \sqrt{10\times4\times4\times2}$ sqcm

$= 17.8$ sqcm

25. Find the area of an isosceles triangles, the measure of one of its equal sides being 10 cm and the third side is 6 cm.

Ans: $S = \frac{10+10+6}{2} = \frac{26}{2} = 13$ cm

$\therefore$ Area if triangle $= \sqrt{13(13-5)(13-5)(13-6)}$ sqcm

$= \sqrt{13\times3\times3\times7}$ sqcm

$= 3\sqrt{91}$ sqcm

26. Find the area of equilateral triangle the length of one of its sides being 24 cm.

Ans: Let, $a = b = c = 24$ cm

$S = \frac{24+24+24}{2}$ cm

$= \frac{72}{2}$ cm

$= 36$ cm

$\therefore$ Area of triangle $= \sqrt{36(36-24)(36-24)(36-24)}\,sqcm$

$= 246.12\text{sqcm}$

27. Find the perimeter and area of a triangle whose sides are $3\text{ cm}, 4\text{ cm}$ **and** 10 cm?

Ans: Perimeter $= 3 + 4 + 5$

$= 12\text{ cm}$

$\therefore \text{S} =$ semi perimeter $= \frac{12}{2}$

$\text{Or} = 6\text{ cm}$

Area of triangle $= \sqrt{6(6-3)(6-4)(6-5)}\text{sqcm}$

$= 6\text{sqcm}$

28. Using Heron's formula, find area of triangle whose sides are $6\text{ cm}, 8\text{ cm}$ **and** 10 cm?

Ans: Find the area of the triangle,

$S = \frac{6+8+10}{2}$

$= \frac{24}{2}$

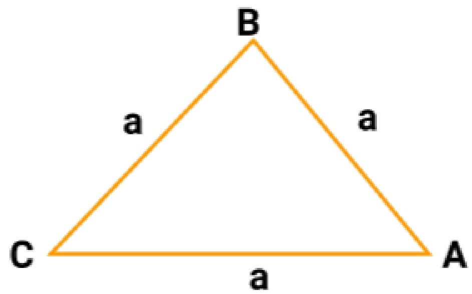
$= 12\text{ cm}$

Area of triangle $= \sqrt{12(12-6)(12-8)(12-10)}\text{sqcm}$

$= 24\text{sqcm}.$

3 Marks Questions:

1. A traffic signal board, indicating 'SCHOOLAHEAD' is an equilateral triangle with side 'a'. Find the area of the signal board, using Heron's formula. If its perimeter is 180 cm**, what will be the area of the signal board?**



Ans: Let the Traffic signal board is ΔABC. According to question, Semi-perimeter of $\Delta ABC(s) = \frac{a+a+a}{2} = \frac{3a}{2}$ Using Heron's Formula, Area of triangle $ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{\frac{3a}{2}\left(\frac{3a}{2}-a\right)\left(\frac{3a}{2}-a\right)\left(\frac{3a}{2}-a\right)}$$

$$= \sqrt{\frac{3a}{2} \times \frac{a}{2} \times \frac{a}{2} \times \frac{a}{2}}$$

$$= \sqrt{3\left(\frac{a}{2}\right)^4}$$

$$= \frac{\sqrt{3}a^2}{4}$$

Now, Perimeter of this triangle $= 180\text{ cm}$

$\Rightarrow$ Side of triangle $(a) = \frac{180}{3} = 60\text{ cm}$

$\Rightarrow$ Semi-perimeter of this triangle $= \frac{180}{2} = 90\text{ cm}$

Using Heron's Formula, Area of this triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

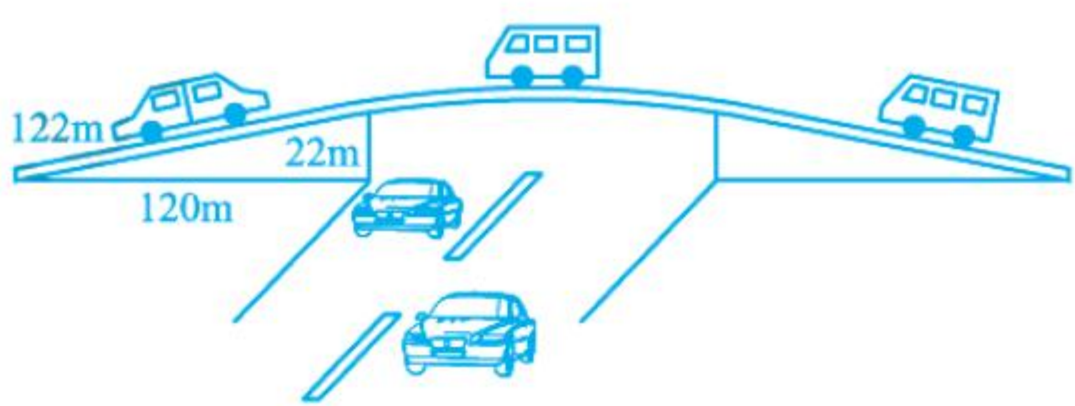
$=\sqrt{90(90-60)(90-60)(90-60)}$

$=\sqrt{90\times 30\times 30\times 30}$

$=30\times 30\sqrt{3}$

$=900\sqrt{3}\text{ cm}^2$

2. The triangular side walls of a flyover has been used for advertisements. The sides of the walls are $122\text{ m}, 22\text{ m}$ and 120 m (see figure). The advertisement yield an earning of $Rs.\ 5000/\text{m}^2$ per year. A company hired one of its walls for 3 months, how much rent did it pay?



Ans: Given: $a=122\text{ m}, b=22\text{ m}$ and $c=120\text{ m}$

Semi-perimeter of triangle $(s)=\dfrac{122+22+120}{2}$

$=\dfrac{264}{2}$

$=132\text{ m}$

Using Heron's Formula,

Area of triangle $=\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{132(122-132)(132-22)(132-120)}$

$= \sqrt{132\times10\times110\times12}$

$= \sqrt{11\times12\times10\times10\times11\times12}$

$= 10\times11\times12$

$= 1320 \text{ m}^2$

Rent for advertisement on wall for 1 year $= Rs\ 5000/\text{m}^2$

$\therefore$ Rent for advertisement on wall for 3 months for

$1320m^2 = \frac{5000}{12}\times3\times1320 = Rs1650000$

Hence rent paid by company $= Rs1650000$

3. Radha made a picture of an aeroplane with coloured paper as shown in figure. Find the total area of the paper used.

Ans: Area of triangular part I:

Here, Semi-perimeter $(s) = \frac{5+5+1}{2} = 5.5 \text{ cm}$

Therefore, Area $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{5.5(5.5-5)(5.5-5)(5.5-1)}$

$= \sqrt{5.5\times0.5\times0.5\times4.5} = 0.75\sqrt{11}$

$= 0.75\times3.31$

$= 2.4825 \text{ cm}^2$

Area of triangular part II = Length $\times$ Breadth

$= 6.5\times1 = 6.5 \text{ cm}^2$

Area of triangular part III (trapezium): $=\frac{1}{2}(AB+DC)\times AE$

$=\frac{1}{2}(AB+DC)\times\sqrt{AD^2-DE^2}$

$=\frac{1}{2}(1+2)\times\sqrt{1-.025}$

$=\frac{1}{2}\times 3\times\frac{\sqrt{3}}{2}$

$=\frac{3\times 1.732}{4}$

$=1.299\ \text{cm}^2$

Area of triangular parts IV and $V=2\left(\frac{1}{2}\times 1.5\times 6\right)$

$=9\ \text{cm}^2$

$\therefore$ Total area $=2.4825+6.2+1.299+9$

$=19.28\ \text{cm}^2$

4. A rhombus shaped field has green grass for 18 cows to graze. If each side of the rhombus is $30\ \text{m}$ and its longer diagonal is $48\ \text{m}$, grass of how much area of grass field will each cow be getting?

Ans: Here, $AB=BC=CD=DA=30\ \text{m}$ and Diagonal $AC=48\ \text{m}$ which divides the rhombus **ABCD** in two congruent triangle.

$\therefore$ Area of $\Delta\text{ABC}=$ Area of ΔACD

Semi-perimeter of $\Delta\text{ABC}(s)=\frac{30+30+48}{2}=54\ \text{m}$

Now Area of rhombus $\text{ABCD}=$ Area of $\Delta\text{ABC}+$ Area of ΔACD

$=2\text{x}$ Area of $\Delta\text{ABC}[\because$ Area of $\Delta\text{ABC}=$ Area of $\Delta\text{ACD}]$

$= 2\sqrt{s(s-a)(s-b)(s-c)}$ [Using Heron's formula]

$= 2\times\sqrt{54(54-30)(54-30)(54-48)}$

$= 2\times\sqrt{54\times24\times24\times6} = 2\times6\times24$

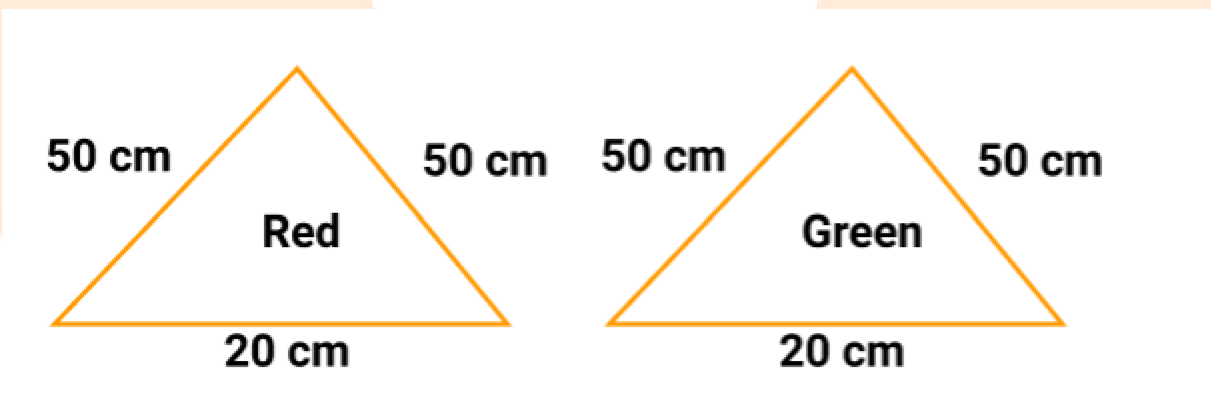
$= 864\text{ m}^2$

$\because$ Field available for 18 cows to graze the grass $= 864m^2$

$\therefore$ Field available for 1 cow to graze the grass $= \frac{864}{18} = 48\text{ m}^2$

5. An umbrella is made by stitching 10 triangular pieces of cloth of two different colours (see figure), each piece measuring $20\text{ cm}, 50\text{ cm}$ **and** 50 cm. **How much cloth of each colour is required for the umbrella?**

Ans: Here, sides of each of 10 triangular pieces of two different colours are $20\text{ cm}, 50\text{ cm}$ and 50 cm.



Semi-perimeter of each triangle $(s) = \frac{20+50+50}{2} = 60\text{ cm}$

Now, Area of each triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{60(60-20)(60-50)(60-50)}$

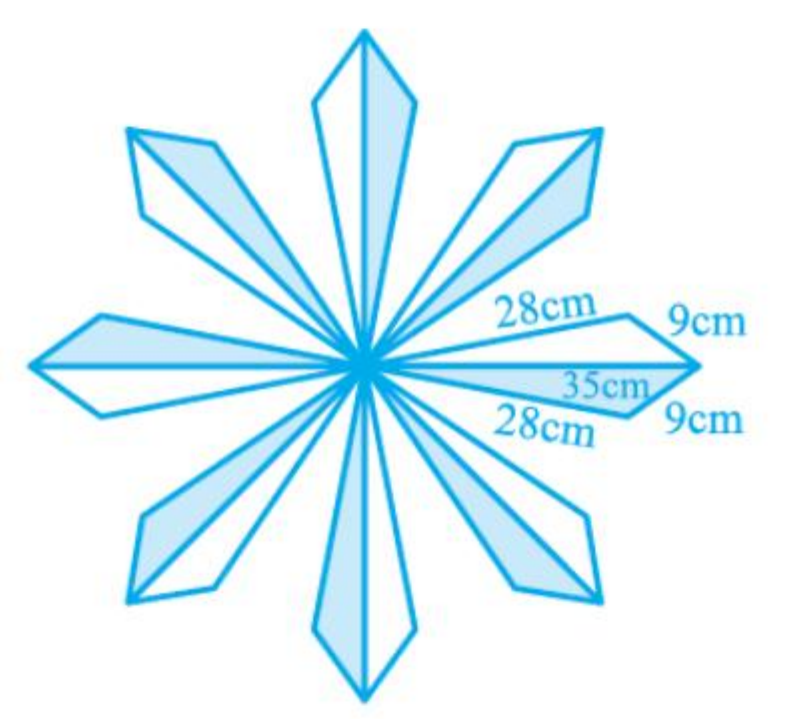
$= \sqrt{60\times40\times10\times10} = 200\sqrt{6}\text{ cm}^2$

According to question, there are 5 pieces of red colour and **5** pieces of green colour.

Cloth required for **5** red pieces $= 5 \times 200\sqrt{6} = 1000\sqrt{6}\ \text{cm}^2$

And Cloth required to **5** green pieces $= 5 \times 200\sqrt{6} = 1000\sqrt{6}\ \text{cm}^2$

6. A floral design on a floor is made up of 16 tiles which are triangular, the sides of the triangle being $9\ \text{cm}, 28\ \text{cm}$ and $35\ \text{cm}$ (see figure). Find the cost of polishing the tiles at the rate of 50 paise per cm^2.



Ans: Here, Sides of a triangular shaped tile area $9\ \text{cm}, 28\ \text{cm}$ and $35\ \text{cm}$.

Semi-perimeter of tile $(s) = \dfrac{9+28+35}{2} = 36\ \text{cm}$

Area of triangular shaped tile $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{36(36-9)(36-28)(36-35)}$

$= \sqrt{36 \times 27 \times 8 \times 1} = 36\sqrt{6}$

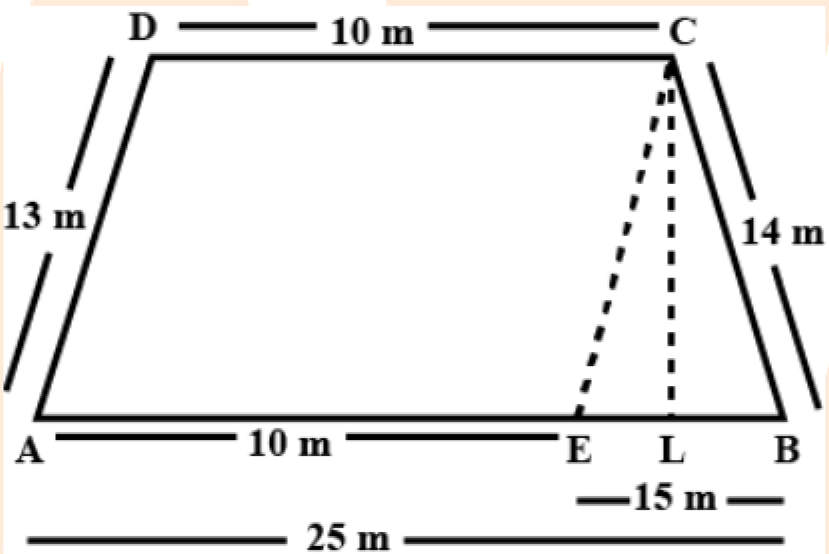
$= 36 \times 2.45 = 88.2 \text{ cm}^2$ (approx.)

$\therefore$ Area of 16 such tiles $= 16 \times 88.2 = 1411.2 \text{ cm}^2$ (Approx.)

$\because$ Cost of polishing 1 cm^2 of tile $=$ $Rs.0.50$

$\therefore$ Cost of polishing 1411.2 cm^2 of tile $=$ $Rs.\,0.50 \times 1411.2 = Rs.\,705.60$ (Approx.)

7. A field is in the shape of a trapezium whose parallel sides are 25 m and 10 m. The non-parallel sides are 14 m and 13 m. Find the area of the field.



Ans: Let ABCD be a trapezium with,

$\text{AB} \parallel \text{CD}$

$\text{AB} = 25 \text{m}$

$\text{CD} = 10 \text{m}$

$\text{BC} = 14 \text{m}$

$\text{AD} = 13 \text{m}$

Draw CE $\parallel$ DA. So, ADCE is a parallelogram with,

$\text{CD} = \text{AE} = 10 \text{m}$

$CE = AD = 13\text{m}$

$BE = AB - AE = 25 - 10 = 15\text{m}$

In ΔBCE, the semi perimeter will be,

$$s = \frac{a+b+c}{2}$$

$$s = \frac{14+13+15}{2}$$

$$s = 21\text{m}$$

Area of ΔBCE,

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{21(21-14)(21-13)(21-15)}$$

$$= \sqrt{21(7)(8)(6)}$$

$$= \sqrt{7056}$$

$$= 84\text{m}^2$$

Also, area of ΔBCE is,

$$A = \frac{1}{2} \times \text{ base } \times \text{ height}$$

$$84 = \frac{1}{2} \times 15 \times CL$$

$$\frac{84 \times 2}{15} = CL$$

$$CL = \frac{56}{5}\text{m}$$

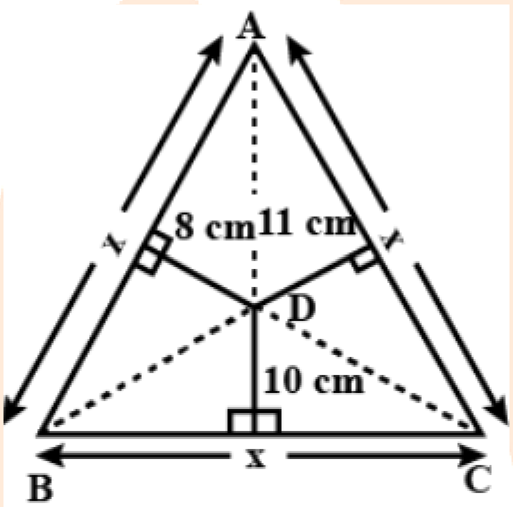
Now, the area of trapezium is,

$$A = \frac{1}{2} \times (25+10)\left(\frac{56}{5}\right)$$

$A = 196\,m^2$

Therefore, the area of the trapezium is $196\,m^2$.

8. From a point in the interior of an equilateral triangle perpendiculars drawn to the three sides are $8\,cm, 10\,cm$ and $11\,cm$ respectively. Find the area of the triangle to the nearest cm. (use $\sqrt{3} = 1.73$)



Ans: Let x be the side of an equilateral triangle.

Therefore, its area $= \frac{\sqrt{3}}{4}x^2$

Also, area $ABC = ar(\Delta ADB) + ar(\Delta BDC) + ar(\Delta CDA)$

$= \frac{1}{2} \times x \times 8 + \frac{1}{2} \times x \times 10 + \frac{1}{2} \times x \times 11$

$= \frac{1}{2} x \times (8 + 10 + 11) = 14.5x$

Again, both areas are equal

$\because \frac{\sqrt{3}}{4}x^2 = 14.5x$

$\Rightarrow x = \frac{58}{\sqrt{3}} \quad \ldots[\because x \neq 0]$

Therefore, area of the equilateral triangle $= \frac{\sqrt{3}x^2}{4} = \frac{\sqrt{3}}{4} \times \left(\frac{58}{\sqrt{3}}\right)^2 = \frac{\sqrt{3}}{4} \times \frac{58 \times 58}{3} \sim 486\,\text{cm}^2$

9. A parallelogram, the length of whose side is $60\,\text{m}$ and $25\,\text{m}$ has ane diagonal $65\,\text{m}$ long. Find the area of the parallelogram.

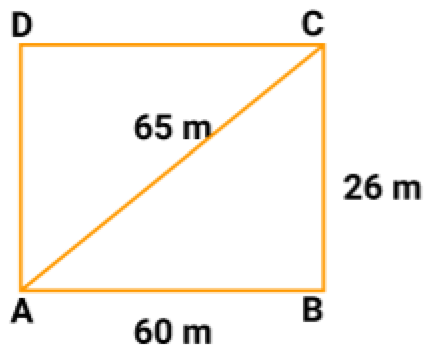
Ans: Let, **AB = DC = 60 cm**

BC = AD = 25m

and $AC = 65m$

Area of parallelogram $\text{ABCD} =$ Area of $\Delta\text{ABC} +$ area of ΔACD

$= 2$ Area of $\Delta\text{ABC}[\therefore ar\Delta\text{ABC} = ar - \Delta\text{ABD}]$



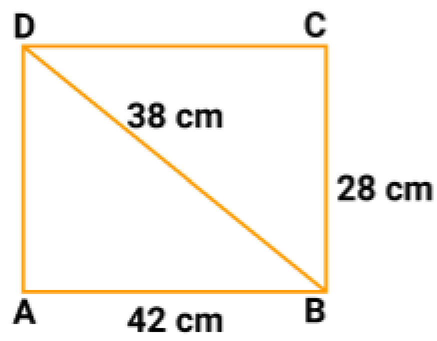
Now area of ....(II)

From (i) and (ii), we get

Area of

$\text{Parallelogram}\text{ABCD} = 2 \times 750 = 15000\text{sqm}$.

10. A parallelogram, the measures of whose adjacent sides are $28\,\text{cm}$ and $42\,\text{cm}$, has one diagonals $38\,\text{cm}$. Find Its altitude on the side $42\,\text{cm}$.



Ans: $AB = DC = 42\text{ cm} = C$

$BC = AD = 28\text{ cm} = b$

And $BD = 38\text{ cm} = a$

Let A be the area of ΔABD

Now, $S = \dfrac{38+28+42}{2} = 54\text{ cm}$

$A = \sqrt{54(54-38)(58-28)(54-42)}$

$= \sqrt{54\times 16\times 26\times 12}\text{sqcm.}$

$= 144\sqrt{13}\text{sqcm}$

Area of $\Delta ABD = 144\sqrt{13}\text{sqcm}$

Again area of $\Delta ABD = \dfrac{1}{2}$ base $\times$ altitude

$= \dfrac{1}{2}\times 42\times h\text{sqcm}$, where $h\text{cm}$ is altitude

$= 21\text{ hsqcm}$

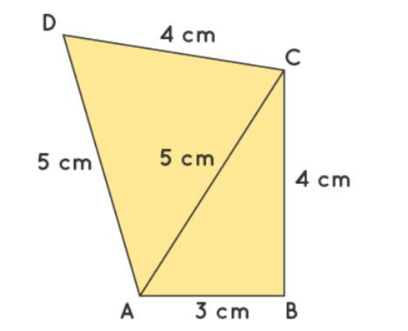
From (i) and (ii), we get,

$21h = 144\sqrt{13}$

$h = \frac{144\sqrt{13}}{21} = \frac{48\sqrt{13}}{7}$ cm

Thus, required altitude $= \frac{48\sqrt{13}}{7}$ cm.

12. Find the area of a quadrilateral ABCD in which $AB = 3\text{ cm}, BC = 4\text{ cm}, CD = 4\text{ cm}$**,** $DA = 5\text{ cm}$ **and** $AC = 5\text{ cm}$



Ans: For $\triangle ABC$, consider

$AB^2 + BC^2 = 3^2 + 4^2 = 25 = 5^2$

$\Rightarrow 5^2 = AC^2$

Since $\triangle ABC$ obeys the Pythagoras theorem, we can say $\triangle ABC$ is rightangled at B.

Therefore, the area of $\triangle ABC = 1/2 \times$ base $\times$ height

$= 1/2 \times 3\text{cm} \times 4\text{cm} = 6\text{cm}^2$

Area of $\triangle ABC = 6\text{cm}^2$

Now, In $\triangle ADC$

we have $a = 5\text{cm}, b = 4\text{cm}$ and $c = 5\text{cm}$

Semi Perimeter: $s =$ Perimeter $/2$

$s = (a+b+c)/2$

$s = (5+4+5)/2$

$s = 14/2$

$s = 7\,\text{cm}$

By using Heron's formula,

Area of $\triangle \text{ADC} = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{7(7-5)(7-4)(7-5)}$

$= \sqrt{7\times 2\times 3\times 2}$

$= 2\sqrt{21}\text{cm}^2$

Area of $\triangle \text{ADC} = 9.2\,\text{cm}^2$ (approx.)

Area of the quadrilateral $ABCD =$ Area of $\triangle ADC +$ Area of $\triangle ABC$

$= 9.2\,\text{cm}^2 + 6\,\text{cm}^2$

Thus, the area of the quadrilateral ABCD is $15.2\,\text{cm}^2$.

14. The perimeter of a triangle is 450 m and its sides are in the ratio of 13:12:5. Find the area of the triangle.

Ans: Let the sides of the triangle be $13\text{x}, 12\text{x}$ and 5x

Perimeter of a triangle $= 450\text{ m}$

$\therefore 13x + 12x + 5x = 450\text{ m}$

or $30\text{x} = 450$

$\therefore \text{x} = 15$

$\therefore$ The sides are $13\times 15, 12\times 15$, and 5×15

I.e. $195\text{ m}, 180\text{ m}$ and 75 m

$\therefore \mathrm{S} = \frac{a+b+c}{2} = \frac{450}{2} = 225\ \mathrm{m}$

$\therefore$ Area of the triangle $= \sqrt{s(s-a)(s-b)(s-c)}$ sqm

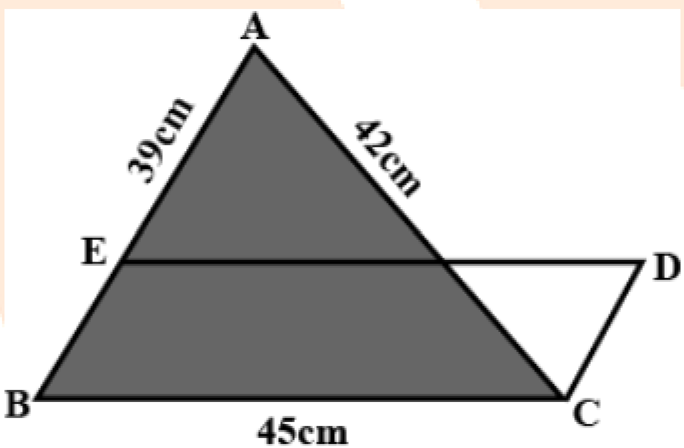
$= \sqrt{225(225-195)(225-180)(225-75)}\,\mathrm{sqm}$

$= \sqrt{225 \times 30 \times 45 \times 150}\,\mathrm{sqm}$

$= (15 \times 15 \times 2 \times 3 \times 5)\,\mathrm{sqm}$

$= 6750\,\mathrm{sqm}$.

15. The sides of a triangle are $39\ \mathrm{cm}, 42\ \mathrm{cm}$ and $45\ \mathrm{cm}$. A parallelogram stands on the greatest side of the triangle and has the same area as that of the triangle. Find the height of the parallelogram.



Ans: To find the area of $\Delta \mathrm{ABC}$

$S = \frac{45+42+39}{2}\ \mathrm{cm}$

$= 63\ \mathrm{cm}$

Therefore, Area of $\Delta \mathrm{ABC} = \sqrt{63(63-45)(63-42)(63-39)}\,\mathrm{sqcm}$

$= \sqrt{63 \times 18 \times 21 \times 24}\,\mathrm{sqcm}$

$= 9 \times 7 \times 2 \times 3 \times 2 \text{sqcm}$

$= 756 \text{sqcm}$

Let h be the height of the parallelogram

Now,

Area of parallelogram $\mathrm{BCDE} =$ Area of $\Delta \mathrm{ABC}$

$\therefore \mathrm{h} \times \mathrm{BC} = 756$

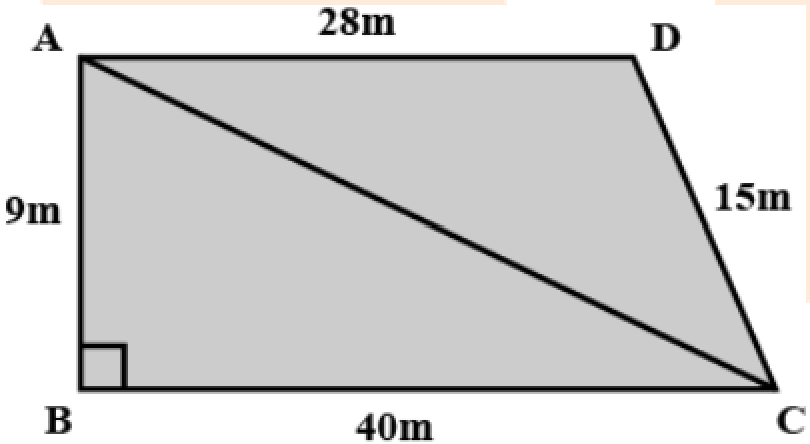
or $45\mathrm{h} = 756$

$\mathrm{h} = \frac{756}{45}$

$\mathrm{h} = 16.8 \text{ cm}$

Hence, height of the parallelogram $= 16.8 \text{ cm}$

16. The students of a school staged a rally for cleanliness campaign. They walked through the lanes in two groups. One group walked through the lanes AB, BC and CA whiles the other group through the lanes AC, CD and DA [fig1.1]. Then they cleaned the area enclosed within their lanes. If $\mathrm{AB} = 9 \text{ m}, \mathrm{BC} = 40 \text{ m}, \mathrm{CD} = 15 \text{ m}, \mathrm{DA} = 28 \text{ m}$ and $\angle B = 90^\circ$, which group cleaned more area and by how much? Find also the total area cleaned by the students.



Ans: We have, right angle $\Delta \mathrm{ABC}$

$AC^2 = AB^2 + BC^2$

$AC^2 = 9^2 + 40^2$

$AC^2 = 1681$

$\therefore AC = 41$

The first group has to clean the area of

ΔABC

which is right angled triangle

Now,

Area of $\Delta ABC = \frac{1}{2} \times 40 \text{ m} \times 9 \text{ m}$

$= 180 \text{sqm}$

The second group has to clean the area of ΔACD which has $AD = 28 \text{ m}$,

$DC = 15 \text{ m}$ and $AC = 41$

Hence,

$S = \frac{28 + 15 + 41}{2}$

$= 42 \text{ m}$

$\therefore$ Area of $\Delta ACD = \sqrt{42(42-28)(42-15)(42-41)} \text{sqm}$

$= \sqrt{42 \times 14 \times 27 \times 1} \text{sqm}$

$= \sqrt{7 \times 3 \times 2 \times 7 \times 2 \times 9 \times 3} \text{sqm}$

$= 126 \text{sqm}$

$\therefore$ First group cleaned more $= (180 - 126) \text{sqm}$

$= 54 \text{sqm}$.

Therefore, Total area cleaned by students $=(180+126)\text{sqm}$

$=306\text{sqm}$.

17. A traffic signal board indicating 'school ahead' is an equilateral triangle with side 'a' find the area of the signal board using heron's. Its perimeter is 180 cm**, what will be its** area?

Ans: Find the area of the single board,

$S=\frac{a+a+a}{2}\text{units} \quad =\frac{3a}{2}\text{units}$

$\therefore$ Area of triangle $=\sqrt{\frac{3a}{2}\times\left(\frac{3a}{2}-a\right)\left(\frac{3a}{2}-a\right)\left(\frac{3a}{2}-a\right)}$

$=\sqrt{\frac{3a}{2}\times\frac{a}{2}\times\frac{a}{2}\times\frac{a}{2}}$

$=\frac{a^2}{4}\sqrt{3}\text{squnits}$

Perimeter $=180\text{ cm}$

Thus, each side $=\frac{180}{3}=60\text{ cm}$

Area of signal board $=\frac{\sqrt{3}}{4}(60)^2\text{sqcm}$

$=900\sqrt{3}\text{sqcm}$

18. A parallelogram the length of whose sides are 80 m**, and** 40 m **has one diagonal** 75 m **long. Find the area of the parallelogram?**

Ans: As according to the question, $\text{AB}=\text{DC}=80\text{ cm}$

$\text{BC}=\text{AD}=40\text{ cm}$ and $\text{AC}=75\text{ cm}$

In $\Delta ABC, S = \frac{80+40+75}{2} = 97.5 \text{ cm}$

Area of triangle $ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{97.5(97.5-80)(97.5-40)(97.5-75)} \text{ sqm}$$

$$= \sqrt{97.5 \times 17.5 \times 57.5 \times 22.5} \text{sqm}$$

$$= \sqrt{2207460.94}$$

$$= 1485.75 \text{sqm}$$

Area of parallelogram $ABCD = 2\times$ Area of ΔABC

$$= 2 \times 1485.7$$

$$= 2971.4 \text{sqm}$$

19. The side of a triangular field is $52 \text{ m}, 56 \text{ m}$, and 60 m find the cost of leveling the field $\text{Rs}18/\text{m}$ if a space of 4 cm is to be left for entry gate.

Ans: The side of a triangular field is $52 \text{m}, 56 \text{m}$, and 60m

The cost of leveling is Rs. $18/\text{m}$.

To find:

The total cost of leveling.

Solution:

1) Leveling is done at the boundary of the field so we will find the perimeter of the field first

The perimeter of the field:

$52 + 56 + 60$

168m.

The length which needs to be leveled is $168-4=164\text{m}$ (space of 4m is left for the entry gate)

2) Cost of leveling is 164×18 =Rs. 2952

The total cost of leveling is Rs. 2952

20. A floral design of a floor is made up of 16 tiles which are triangular. The side of the triangle being $9\text{ cm}, 28\text{ cm}$, and 35 cm. find the cost of polishing the tiles, at Rs 50paisa/ sqcm

Ans: For each triangular tile, we have

$$S=\frac{35+28+9}{2}\text{ cm}=36\text{ cm}$$

$\therefore$ Area of Each tile $=\sqrt{s(s-a)(s-b)(s-c)}$

$$=\sqrt{36(36-35)(36-28)(36-9)}\text{sqcm}$$

$$=36\sqrt{6}\text{sqcm}$$

Area of 16 tile $=16\times36\sqrt{6}\text{sqcm}$

Therefore, cost of polishing $=\text{Rs}\left[\frac{1}{2}\times16\times36\sqrt{6}\right]=\text{Rs}\,288\sqrt{6}$

$$=\text{Rs}(288\times2.45)$$

$$=\text{Rs}\,705.60$$

21. The measure of one side of a right triangle is 42 m. If the difference in lengths of Its hypotenuse and other side is 14 cm, find the measure of two unknown side?

Ans: Let $\text{AB}=\text{y}$ and $\text{AC}=\text{x}$ and $\text{BC}=42\text{ cm}$

Therefore, By the given condition,

$x - y = 14$(i)

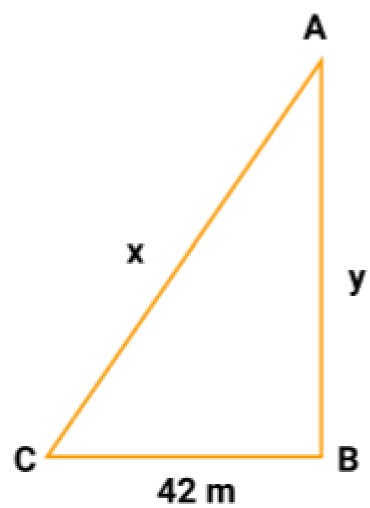
By Pythagoras theorem,

$x^2 - y^2 = 1764$

$(x+y)(x-y) = 1764$

$\therefore 14(x+y) = 1764$ using (ii)

$\therefore x + y = \frac{1764}{14} = 126$ (iii)



Adding (ii) and (iii), we get

$2x = 140$

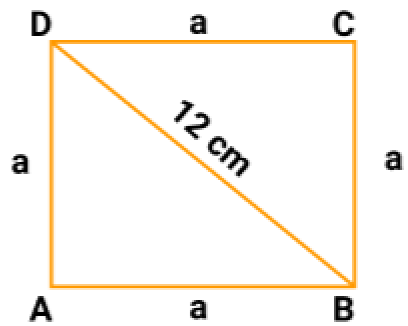
i.e. $x = 70$

$\therefore y = 126 - x$

$y = 126 - 70$

$= 56$

22. The perimeter of a rhombus $ABCD$ is 80 cm. find the area of rhombus if Its diagonal BD measures 12 cm.



Ans: Given that,

Perimeter of rhombus $=80\,\text{m}$

Perimeter of rhombus $=4\times$ side

$\Rightarrow 4\text{a}=80$

$\Rightarrow \text{a}=20\,\text{m}$

Now in ΔABD,

$\therefore \text{S}=\dfrac{20+20+12}{2}=26$

so,

Area of $\Delta\text{ABD}=\sqrt{26\times6\times6\times14}\,\text{sqcm}$

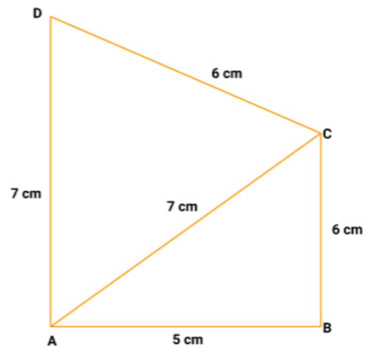
$=114.4\,\text{sqcm}$

Area of rhombus $=2\times$ area of ΔABD

$=2\times114.4\,\text{sqcm}$

$=228.8\,\text{sqcm}$

23. Find area of quadrilateral ABCD **in which** $\text{AB}=5\text{ cm}, \text{BC}=6\text{ cm}, \text{CD}=6\text{ cm}, \text{DA}=7\text{ cm}$**, And** $\text{AC}=7\text{ cm}$



Ans: Area of quadrilateral ABCD = Area of $\Delta ABC +$ Area of ΔACD (i)

In ΔABC,

$$S = \frac{5+6+7}{2} = 9 \text{ cm}$$

Area of $\Delta ABC = \sqrt{9(9-5)(9-6)(9-7)}\text{sqcm}$

$= \sqrt{9 \times 4 \times 3 \times 2}\text{sqcm}$

$= 6\sqrt{6}\text{sqcm}$

$= 14.4\text{sqcm}$

In ΔACD,

$$S = \frac{7+7+6}{2} = 10 \text{ cm}$$

$\therefore$ Area of $\Delta ACD = \sqrt{10(10-7)(10-7)(10-6)}\text{sqcm}$

$= \sqrt{10 \times 3 \times 3 \times 4}\text{sqcm}$

$= 18.9\text{sqcm}$

Area of quadrilateral $ABCD = (14.4 + 18.9)\text{sqcm}$

$= 33.3\text{sqcm}$.

24. Shashi Kant has a vegetable garden in the shape of a rhombus. The length of each side of garden is 35 m And Its diagonal is 42 m long. After growing the

vegetables in it. He wants to divide it in seven equal parts And look after each part once a week. Find the area of the garden which he has to look after daily.

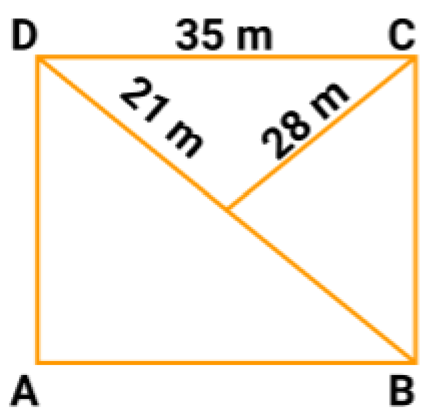
Ans: Let ABCD be garden

$\therefore DC = 35\text{ m}$

$DB = 42\text{ m}$

Draw

$CE \perp DB$



The diagonals of a rhombus bisect each other at right angles.

$\therefore DE = \frac{1}{2} \quad DB = \frac{1}{2} \times 42 \quad$ or 21 m

Now

$CE^2 = CD^2 - DE^2$

$= 35^2 - 21^2$

$= 784$

$CE = 28\text{ m}$

Area of $\Delta DBC, = \frac{1}{2} \times DB \times CE$

$= \frac{1}{2} \times 42 \times 28$

$= 588 \text{sqcm}$

$\therefore$ Area of the garden $ABCD = 2 \times 588 \text{sqm}$

$= 1176 \text{sqm}$

Area of the garden he has to look after, daily $= \frac{1176}{7} \text{sqm}$

$= 168 \text{sqm}$

25. The perimeter of a triangle is 480 meters and its sides are in the ratio of 1:2:3.Find the area of triangle?

Ans: Let the sides of the triangle be **x, 2x, 3x**

Perimeter of the triangle $= 480 \text{ m}$

$\therefore x + 2x + 3x = 480 \text{ m}$

$6x = 480 \text{ m}$

$x = 80 \text{ m}$

Therefore, The sides are $80 \text{ m}, 160 \text{ m}, 240 \text{ m}$

So,

$S = \frac{80 + 160 + 240}{2} = \frac{480}{2}$

$= 240 \text{ m}$

And,

$\therefore$ Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)} \text{sqm}$

$= \sqrt{240(240-80)(240-160)(240-240)} \text{sqm}$

$= 0 \text{sqm}$

Therefore, Triangle doesn't exist with the ratio **1:2:3** whose perimeter is 480 m.

26. Find the cost of leveling the ground in the form of equilateral triangle whose side is 12 m at $\text{Rs } 5$ per square meter.

Ans: Ans: Here, sides are $12 \text{ m}, 12 \text{ m}, 12 \text{ m}$,

$$\therefore S = \frac{12+12+12}{2}$$

$= 18 \text{ cm}$

And,

Area of equilateral triangle $= \sqrt{s(s-a)(s-b)(s-c)} \text{sqm}$

$= \sqrt{18(18-12)(18-12)(18-12)} \text{sqm}$

$= \sqrt{18 \times 6 \times 6 \times 6} \text{sqm}$

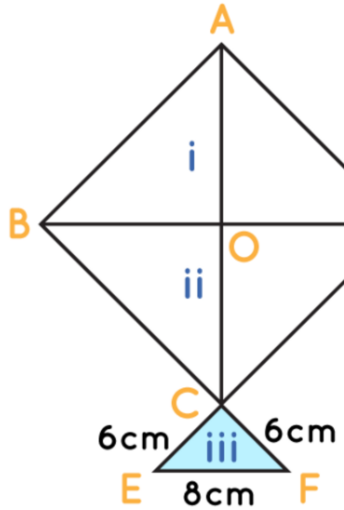
$= \sqrt{6 \times 3 \times 6 \times 6 \times 6} \text{sqm}$

$= 36\sqrt{3} \text{sqm}$

$\therefore$ Cost of leveling ground $= 5 \times 36 \times 1.73$

$= \text{Rs}311.4 \text{ m}$

27. A kite in the shape of a square with diagonal 32 cm and an isosceles triangle of base 8 cm and side 6 cm each is to be made of three different shades. How much paper of each shade has been used in it? (use $\sqrt{5} = 2.24$)



Ans: Let $ABCD$ be the square and ΔCEF be an isosceles triangle.

Let the diagonals bisect each other at **O.**

Then, $AO = \frac{1}{2} \times 32 \text{ cm}$

$= 16 \text{ cm}$

Area of shaded portion $I = \frac{1}{2} \times 16 \times 32 \text{sqcm}$

$= 256 \text{sqcm}$

And,

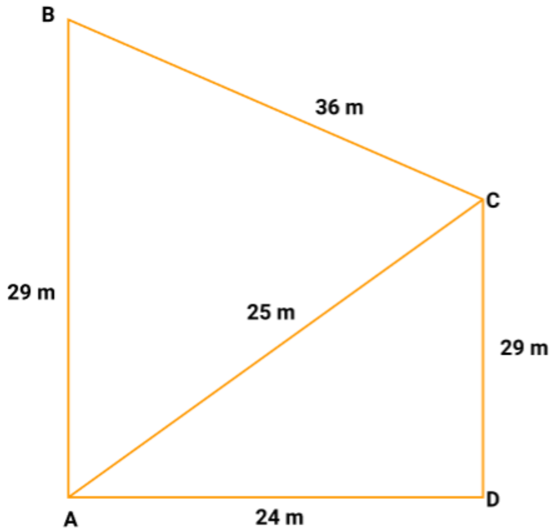
Area of portion III $= \frac{a}{4}\sqrt{4b^2 - a^2} \;\; = \frac{8}{4}\sqrt{4 \times (6)^2 - 8}$

$= 17.92 \text{sqcm}$

Thus, the papers of three shades required are $256 \text{ sqcm}, 256 \text{sqcm}$ and 17.92sqcm.

28. The sides of a quadrangular field, taken in order are $29 \text{ m}, 36 \text{ m}, 7 \text{ m}$ and 24 m

respectively. The angle contained by the last two sides is a right angle. Find its area.



Ans: As the sides are provided in order. Therefore, the length of the last two sides are 7m and 24m respectively.

As we know that:

In a right angle triangle using Pythagoras theorem,

$$(\text{ Hypotenuse })^2 = (\text{ Base })^2 + (\text{ Height })^2$$

Let us assume:

- The diagonal of the field be $\mathbf{d}$.

Substituting the values,

$$d)^2 = (24)^2 + (7)^2$$

$$d)^2 = 576 + 49$$

$$d)^2 = 625$$

$$d = 25$$

Hence,

Diagonal of the park $= 25\text{m}$

Semi-perimeter = Perimeter $/2$

$= (29 + 36 + 25)\text{m} / 2$

$= 90\text{m} / 2$

$= 45\text{m}$

Substituting the values,

Area $= \sqrt{45(45-29)(45-36)(45-25)}$

$= \sqrt{45 \times 16 \times 9 \times 20}$

$= \sqrt{129600}$

$= 360$

Finding area of 2nd triangle:

1st side $= 7\text{m}$

2nd side $= 24\text{m}$

3rd side $= 25\text{m}$

Finding semi-perimeter of the triangle:

Semi-perimeter $=$ Perimeter/2

$= (7 + 24 + 25)\text{m} / 2$

$= 56\text{m} / 2$

$= 28\text{m}$

Finding area of the 2nd triangle using Heron's formula:

Area $= \sqrt{28(28-7)(28-24)(28-25)}$

$= \sqrt{28 \times 21 \times 4 \times 3}$

$= \sqrt{7056} = 84$

Hence, area of the 2 nd triangle is $84\,\text{m}^2$.

Finding area of the quadrangular field:

Ar. of the field $=($ Ar. of 1 st $\Delta)+($ Ar. of 2 nd Δ $)$

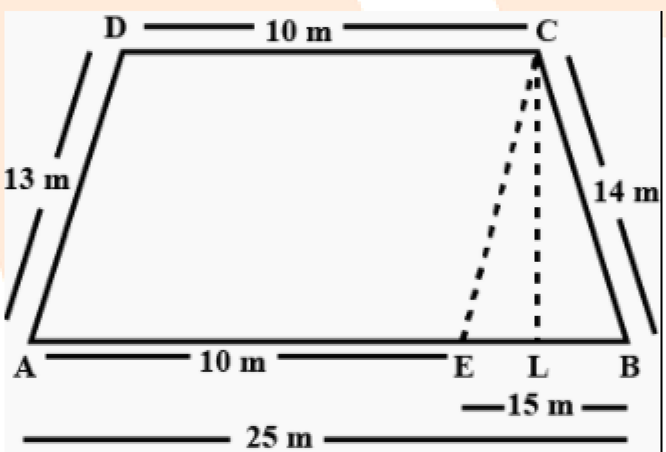
$=(360+84)\text{m}^2$

$=444\,\text{m}^2$

Hence area of the field is $=\mathbf{444}\text{m}^2$

4 Marks Questions

1. A field in the shape of a trapezium whose parallel side are $25\,\text{m}$ and $10\,\text{m}$. The non- parallel side are $14\,\text{m}$ and $13\,\text{m}$. Find the area of the field.



Ans: Let $ABCD$ be a trapezium with,

$AB \| CD$

$AB = 25\text{m}$

$CD = 10\text{m}$

$BC = 14\text{m}$

$AD = 13\text{m}$

Draw CE | DA. So, ADCE is a parallelogram with,

$$CD = AE = 10\,m$$

$$CE = AD = 13\,m$$

$$BE = AB - AE = 25 - 10 = 15\,m$$

In ΔBCE, the semi perimeter will be,

$$s = \frac{a+b+c}{2}$$

$$s = \frac{14+13+15}{2}$$

$$s = 21\,m$$

Area of ΔBCE,

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{21(21-14)(21-13)(21-15)}$$

$$= \sqrt{21(7)(8)(6)}$$

$$= \sqrt{7056}$$

$$= 84\,m^2$$

Also, area of ΔBCE is,

$$A = \frac{1}{2} \times \text{ base } \times \text{ height}$$

$$84 = \frac{1}{2} \times 15 \times CL$$

$$\frac{84 \times 2}{15} = CL$$

$$CL = \frac{56}{5}\,m$$

Now, the area of trapezium is,

$A = \frac{1}{2}$ (sum of parallel sides) (height)

$A = \frac{1}{2} \times (25+10)\left(\frac{56}{5}\right)$

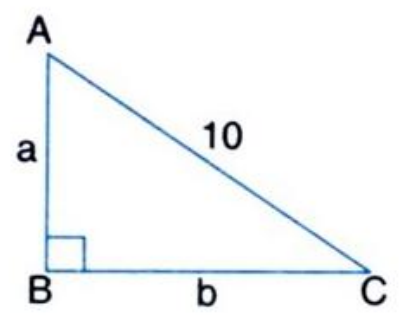
$A = 196\,m^2$

Therefore, the area of the trapezium is $196\,m^2$.

2. The perimeter of a right triangle is 24 cm. If its hypotenuse is 10 cm, find the other two sides. Find its area by using the formula area of a right triangle. Verify your result by using Heron's formula.

Ans: Let the sides of right Δ be 'a' cm and 'b' cm.

Then,



$a+b+c=24$

$\Rightarrow a+b+10=24$

$\Rightarrow a+b=24-10$

$\Rightarrow a+b=14$……….(1)

$a^2+b^2=(10)^2$

Also, $a^2+b^2=100$

$\Rightarrow a^2+b^2=100......(2)$

We know that

$(a+b)^2=a^2+2ab+b^2$

$(14)^2=100+2ab$

$-2ab=100-196$

$-2ab=-96$

$ab=\frac{96}{2}=48$

$=> ab=48......$

Also,

$(a-b)^2=a^2-2ab+b^2$

$(a-b)^2=100-2\times 48$

$(a-b)^2=100-96$

$(a-b)^2=4$

$(a-b)=\sqrt{4}=2$

$\Rightarrow (a-b)=2$

Solving (1) and (4) we get;

$\therefore a=8$ and $b=6$

Now,

$\frac{a+b+c}{2}$

$\mathrm{S}=\frac{24}{2}=12$

Area of $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$\Rightarrow \sqrt{12(12-8)(12-6)(12-10)}$

$=> \sqrt{12\times4\times6\times2}$

$=> \sqrt{2\times2\times3\times2\times2\times2\times3\times2}$

$=> \sqrt{2^2\times2^2\times2^2\times3^2}$

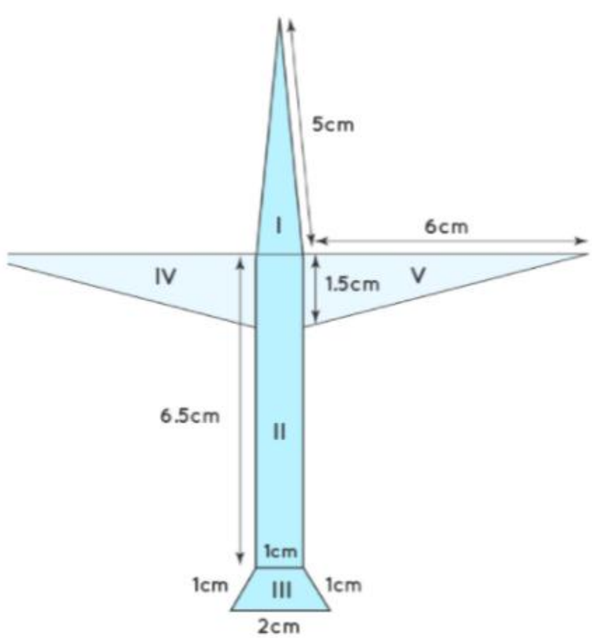
$=> 2\times2\times2\times3$

$=> 24$

Hence,

The area of Δ is 24cm.

3. Radha made a picture of an aero plane with colored paper as shown in fig. find the total area of the paper used.



Ans: Area (1) $=$area of is iosceles triangle with $a = 1\text{ cm}$ and $b = 5\text{ cm}$

$$= \frac{a}{4}\sqrt{4b^2 - a^2}$$

$$= \frac{1}{4}\sqrt{100-1} = \frac{\sqrt{99}}{4}\text{sq cm(approx)}$$

Area (ii) = area of rectangle with

$L = 6.5\text{ cm}$ and $b = 1\text{ cm}$

$$= 6.5 \times 1\text{sqcm}$$

$$= 6.5\text{sqcm}$$

Area (iii) = Area of trapezium

$= 3\times$ Area of equilateral Δ with side $= 1\text{ cm}$

$$= 3 \times \frac{\sqrt{3}}{4} \times (1)^2\text{sqcm}$$

$$= \frac{3 \times 1.732}{4} \text{ or } \frac{5.196}{4}\text{sqcm}$$

$= 1.3\text{sqcm}$ (approx.)

Area of $(\text{IV} + \text{V}) = 2 \times \frac{1}{2} \times 6 \times 1.5\text{sqcm} = 9\text{sqcm}$

Total area of the paper used = Area $(\text{I} + \text{II} + \text{III} + \text{IV} + \text{V})$

$$= (2.5 + 6.5 + 1.3 + 9)\text{sqcm}$$

$$= 19.3\text{sqcm}$$