

## Physics JEE (Main and Advance) Formula

### Unit and dimensions

Unit:

Measurement of any physical quantity is in terms of an internationally accepted certain basic standard called unit.

- Fundamental units:

S.No.	Physical quantity	SI unit	Symbol
1	Length	Metre	m
2	Mass	Kilogram	Kg
3	Time	Second	S
4	Electric current	Ampere	A
5	Temperature	Kelvin	K
6	Luminous intensity	Candela	Cd
7	Amount of substance	Mole	mol

- Supplementary units:

S.No.	Physical quantity	SI unit	Symbol
1	Plane angle	Radian	R
2	Solid angle	Steradian	Sr

- Metric prefixes:

S.No.	Prefix	Symbol	Value
1	Centi	c	$10^2$
2	Mili	m	$10^3$
3	Micro	$\mu$	$10^6$
4	Nano	n	$10^9$
5	Pico	p	$10^{12}$
6	Kilo	K	$10^3$
7	Mega	M	$10^6$

### Rectilinear motion

- Average velocity (in an interval):  $V_{av} = \frac{\text{Total displacement}}{\text{total time taken}} = \frac{\vec{r}_f - \vec{r}_i}{\Delta t}$
- Average speed (in an interval): Average speed =  $\frac{\text{Total displacement}}{\text{total time taken}}$
- Instantaneous velocity (at an instant):  $V_{inst} = \lim_{\Delta t \rightarrow 0} \frac{\Delta r}{\Delta t}$
- Average acceleration (in an interval):  $= \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t}$
- Instantaneous acceleration (at an instant):  $a = \frac{dv}{dt} = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t}$

Graphs in uniformly accelerated motion along a straight line (a = 0)

x is a quadratic polynomial in terms of t. Hence x - t graph is a parabola.

v is a linear polynomial in terms of t. Hence v - t graph is a straight line of slope a.

a – t graph is a horizontal line because a is constant.

➤ Maxima & minima

$$\frac{dy}{dx} = 0 \text{ \& } \frac{d}{dx} \frac{d}{dx} < 0 \text{ at maximum and } \frac{dy}{dx} = 0 \text{ \& } \frac{d}{dx} \frac{d}{dx} > 0 \text{ at minima.}$$

Equations of motion (for constant acceleration)

a)  $v = u + at$

b)  $s = ut + \frac{1}{2}at^2, s = vt - \frac{1}{2}at^2, x_t = x, +ut + \frac{1}{2}at^2$

c)  $v^2 = u^2 + 2as$

d)  $s = \frac{(u+v)}{2}t$

e)  $S_n = u + \frac{a}{2}(2n-1)$

For freely falling bodies: ( $u = 0$ )

(Taking upward direction as positive)

a)  $V = gt$

b)  $s = \frac{1}{2}gt^2, s = vt + \frac{1}{2}gt^2, h_f = h, \frac{1}{2}gt^2$

c)  $v^2 = 2gs$

d)  $S_n = \frac{g}{2}(2n-1)$

## Projectile motion & vectors

Time of flight:  $T = \frac{2u \sin \theta}{g}$

Horizontal range:  $R = \frac{u^2 \sin 2\theta}{g}$

Maximum height:  $H = \frac{u^2 \sin^2 \theta}{2g}$

Trajectory equation (equation of path):

$$y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta} = x \tan \theta \left(1 - \frac{x}{R}\right)$$

Projection on an inclined plane

Range	Up the incline	Down the incline
	$\frac{2u^2 \sin \alpha \cos(\alpha + \beta)}{g \cos^2 \beta}$	$\frac{2u \sin^2 \alpha \cos(\alpha - \beta)}{g \cos^2 \beta}$
Time of flight	$\frac{2u \sin \alpha}{g \cos \beta}$	$\frac{2u \sin \alpha}{g \cos \beta}$
Angle of projection with incline plane for maximum range	$\frac{\pi}{4} - \frac{\beta}{2}$	$\frac{\pi}{4} + \frac{\beta}{2}$
Maximum range	$\frac{u^2}{g(1 + \sin \beta)}$	$\frac{u^2}{g(1 - \sin \beta)}$

### Relative motion

❖  $V_{AB}$  (Velocity of A with respect to B) =  $V_A - V_B$

❖  $a_{AB}$  (Acceleration of A with respect to B) =  $a_A - a_B$

❖ Relative motion along straight line -  $x_{BA} = x_B - x_A$

➤ CROSSING RIVER

A boat or a man in a river always moves in the direction of resultant velocity of velocity of boat (or man) and velocity of river flow.

1. Shortest time:

Velocity along the river  $V_x = V_R$

Velocity perpendicular to the river  $V_f = V_{mR}$

The net speed is given by  $V_m = \sqrt{V_{mR}^2 + V_R^2}$

2. Shortest path:

Velocity along the river  $V_x = 0$

And velocity perpendicular to the river  $V_y = \sqrt{V_{mR}^2 - V_R^2}$

The net speed is given by  $V_m = \sqrt{V_{mR}^2 - V_R^2}$

at an angle of  $90^\circ$  with the river direction.

Velocity  $V_y$  used only to cross the river

therefore, time to cross the river  $t = \frac{d}{v_y} = \frac{d}{\sqrt{v_{mR}^2 - v_R^2}}$

and velocity  $V_x$  is zero therefore, in this case the drift should be zero.

$$V_r = V_{mR} \sin \theta = 0 \text{ or } V_R = V_{mR} \sin \theta \text{ or } \theta = \sin^{-1} \frac{V_R}{V_{mR}}$$

➤ RAIN PROBLEMS

$$v_{RM} = \vec{v}_R v_m \text{ or } V_{Rm} = \sqrt{v_R^2 + v_m^2}$$

## Newton's laws of motion

1. From 3<sup>rd</sup> law of motion

$$\vec{F}_{AB} = -\vec{F}_{BA} \quad \vec{F}_{AB} = \text{Force on A due to B} \quad \vec{F}_{BA} = \text{Force on B due to A}$$

2. From second law of motion

$$F_x = \frac{dP_x}{dt} = ma_x, F_y = \frac{dP_y}{dt} = ma_y$$

5. Weighing machine

A weighing machine does not measure the weight but measures the force exerted by object on its upper surface.

6. Spring force  $F = -kx$

$x$  is displacement of the free end from its natural length or deformation of the spring where  $K =$  spring constant.

7. Spring property  $K \ell = \text{constant}$

$\ell =$  natural length of spring

8. If spring is cut into 2 in the ratio  $m : n$  then spring constant is given by

$$\ell_1 = \frac{m\ell}{m+n}; \ell_2 = \frac{n\ell}{m+n} \quad k\ell = k_1\ell_1 = k_2\ell_2$$

For series combination of springs:  $\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} + \dots$

For parallel combination of spring  $k_{eq} = k_1 + k_2 + k_3 + \dots$

9. Spring balance:

It does not measure the weight. It measures the force exerted by the object at the hook.

$$11. \quad a = \frac{(m_2 - m_1)g}{m_1 + m_2}$$
$$T = \frac{2m_1m_2g}{m_1 + m_2}$$

12. Wedge constraint:

Components of velocity along perpendicular direction to the contact plane of the 2 objects equal if there is no deformations and they remain in contact.

13. Newton's law for a system

$$\vec{F}_{ext} = m_1a_1 + m_2a_2 + m_3a_3 + \dots$$

$F_{ext}$  = Net external force on the system

$m_1, m_2, m_3$  are the masses of the objects of the system and  $a_1, a_2, a_3$  are the acceleration of the objects respectively.

14. Newton's law for non-inertial frame:

$$\vec{F}_{Real} + \vec{F}_{Pseudo} = m\vec{a}$$

Net sum of real and pseudo force is taken in the resultant force.

$a$  = acceleration of the particle in the non-inertial frame

- Inertial reference frame: frame of reference moving with constant velocity
- Non inertial reference frame: a frame of reference moving with non-zero acceleration.

## Friction

Friction force is of 2 types

- a) Kinetic
- b) Static

Kinetic friction:  $f_k = \mu_k N$

The proportionality constant  $\mu_k$  is called the coefficient of kinetic friction and its value depends on the nature of the 2 surfaces in contact.

Static friction:

It exists between the 2 surfaces when there is tendency of relative motion but no relative motion along the 2 contact surfaces.

This means static friction is a variable and self adjusting force however it has a maximum value called limiting friction.

$$f_{\max} = \mu_s N$$

$$0 \leq f_s \leq f_{s\max}$$

### Work, power & energy

- Work done by constant force:  $W = F \cdot \vec{S}$
- Work done by multiple forces:

$$\Sigma \vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \dots$$

$$W = [\Sigma F] \cdot S$$

$$W = F_1 \cdot \vec{S} + F_2 \cdot \vec{S} + F_3 \cdot \vec{S} + \dots$$

$$\text{or } W = W_1 + W_2 + W_3 + \dots$$

- Work done by a variable force

$$dW = \vec{F} \cdot d\vec{s}$$



- Relation between momentum and kinetic energy

$$K = \frac{p^2}{2m} \text{ and } P = \sqrt{2mK}; P = \text{linear momentum}$$

- Potential energy

$$\int_{U_1}^{U_2} dU = -\int_{r_1}^{r_2} \vec{F} \cdot d\vec{r} \text{ i.e., } U_2 - U_1 = -\int_{r_1}^{r_2} \vec{F} \cdot d\vec{r} = -W$$

$$U = -\int_{\infty}^r \vec{F} \cdot d\vec{r} = -W$$

- Conservative forces  $F = U/r$
- Work energy theorem  $W_C + W_{NC} + W_{RS} = \Delta K$
- Modified form of work energy theorem

$$W_C = -\Delta U$$

$$W_{NC} + W_{PS} = \Delta K + \Delta U$$

$$W_{NC} + W_{PS} = \Delta E$$

- Power

The average power ( $\bar{P}$  or  $p$ ) delivered by an agent is given by  $\bar{P}$  or  $p = \frac{W}{t}$

$$P = \frac{FdS}{dt} = F \frac{dS}{dt} = \vec{F} \cdot \vec{v}$$

## Circular motion

1. Average angular velocity  $\omega = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$
2. Instantaneous angular velocity  $\omega = \frac{d\theta}{dt}$
3. Average angular exploration  $\alpha_{av} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}$

4. Instantaneous angular acceleration  $\alpha = \frac{d\omega}{dt} = \omega \frac{d\omega}{d\theta}$

5. Relation between speed and angular velocity  $v = r\omega$  and  $v = \omega r$

7. Tangential acceleration (rate of change of speed)  $a_t = \frac{dv}{dt} = r \frac{d\omega}{dt} = \omega \frac{dr}{dt}$

8. Radial or normal or centripetal acceleration  $a_r = \frac{v^2}{r} = \omega^2 r$

9. Total acceleration

$$a = a_t + a_r \quad a = (a_t^2 + a_r^2)^{1/2}$$

Where  $a_t = \alpha r$  and  $a_r = \omega v$

10. Angular exploration

$$\alpha = \frac{d\omega}{dt}$$

12. Radius of curvature R  $R = \frac{v^2}{a_{\perp}} = \frac{mv^2}{F_{\perp}}$

If y is a function of x. i.e.  $y=f(x)$   $R = \frac{1 + \frac{dy^2}{dx^2}}{\frac{d^2y}{dx^2}}$

13. Normal reaction of road on a concave bridge

$$N = mg \cos \theta + \frac{mv^2}{r}$$

#### 14. Normal reaction on a convex bridge

$$N = mg \cos \theta - \frac{mv^2}{r}$$

#### 15. Skidding of vehicle on a level road

$$v_{safe} = \sqrt{\mu gr}$$

#### 16. Skidding of an object on a rotating platform

$$\omega_{max} = \sqrt{\mu g / r}$$

#### 17. Bending of cyclist

$$\tan \theta = \frac{v^2}{rg}$$

#### 18. Banking of road without friction

$$\tan \theta = \frac{v^2}{rg}$$

#### 19. Banking of road with friction

$$\frac{v^2}{rg} = \frac{\mu + \tan \theta}{1 - \mu \tan \theta}$$

#### 20. Maximum also minimum safe speed on a banked fictional road

$$V_{max} = \frac{rg(\mu + \tan \theta)^{1/2}}{(1 - \mu \tan \theta)}$$

$$V_{min} = \frac{rg(\mu - \tan \theta)^{1/2}}{(1 + \mu \tan \theta)}$$

#### 21. Centrifugal force (pseudo force)

$f = m\omega^2 r$ , acts outwards when the particle itself is taken as a frame.

## 22. Effect of Earth's rotation on apparent weight

$$N = mg - mR\omega^2 \cos^2 \theta$$

Where  $\theta$  latitude at a place.

## 23. Various quantities for a critical condition in a vertical loop at different positions (true for a string or on a smooth track)

### 24. Conical pendulum:

$$T \cos \theta = mg$$

$$T \sin \theta = m\omega^2 r$$

$$\text{Time period} = 2\pi \sqrt{\frac{L \cos \theta}{g}}$$

### 25. Relations among Angular variables

$\omega_0$  initial ang. Velocity  $\omega = \omega_0 + \alpha t$

$\omega$  find angular velocity  $\theta = \omega_0 t + \frac{1}{2} \alpha t^2$

$\omega$  const. angular acceleration  $\omega^2 = \omega_0^2 + 2\alpha\theta$

$\theta$  angular displacement

## Centre of mass

➤ Mass moment:  $\vec{M} = m\vec{r}$

- Centre of mass of a system of ‘N’ discrete particles

$$r_{cm} = \frac{m_1 r_1 + m_2 r_2 + \dots + m_n r_n}{m_1 + m_2 + \dots + m_n}; r_{cm} = \frac{\sum_{i=1}^n m_i r_i}{\sum_{i=1}^n m_i} = \frac{1}{M} \sum_{i=1}^n m_i r_i$$

- Centre of mass of a continuous mass distribution

$$x_{cm} = \frac{\int x dm}{dm}, y_{cm} = \frac{\int y dm}{dm}, z_{cm} = \frac{\int z dm}{dm}$$

$dm = M$  (mass of the body)

- **Centre of mass of some common systems**

A system of two-point masses  $m_1 r_1 = m_2 r_2$

The centre of mass lies closer to the heavier mass.

Rectangular plate (by symmetry)

$$x_c = \frac{b}{2}, y_c = \frac{L}{2}$$

A triangular plate (by qualitative argument) at the centroid:

$$y_c = \frac{h}{3}$$

A semi-circular ring

$$y_c = \frac{2R}{\pi}, x_c = O$$

A semi-circular disc

$$y_c = \frac{4R}{3\pi}, x_c = O$$

A hemispherical shell

$$y_c = \frac{R}{2}, x_c = O$$

A solid hemisphere

$$y_c = \frac{3R}{8}, x_c = O$$

A circular cone (solid)

$$y_c = \frac{h}{4}$$

A circular cone (hollow)

$$y_c = \frac{h}{3}$$

➤ **Motion of centre of mass and conservation of momentum:**

Velocity of centre of mass of system

$$v_{cm} = \frac{m_1 \frac{d\vec{r}_1}{dt} + m_2 \frac{d\vec{r}_2}{dt} + m_3 \frac{d\vec{r}_3}{dt} + \dots + m_n \frac{d\vec{r}_n}{dt}}{M} = \frac{m_1 v_1 + m_2 v_2 + m_3 v_3 + \dots + m_n v_n}{M}$$

$$\vec{P}_{system} = M\vec{v}_{cm}$$

➤ **Acceleration of centre of mass of system**

$$a_{cm} = \frac{m_1 \frac{d\vec{v}_1}{dt} + m_2 \frac{d\vec{v}_2}{dt} + m_3 \frac{d\vec{v}_3}{dt} + \dots + m_n \frac{d\vec{v}_n}{dt}}{M} = \frac{m_1 a_1 + m_2 a_2 + m_3 a_3 + \dots + m_n a_n}{M}$$

$$= \frac{\text{Net force on system}}{M} = \frac{\text{Net External Force} + \text{Net internal force}}{M} = \frac{\text{Net External Force}}{M}$$

$$F_{ext} = Ma_{cm}$$

➤ **Impulse**

Impulse of a force F action on a body is defined as:-

$$J = \int_{t_1}^{t_2} F dt \quad J = P \text{ (impulse- momentum theorem)}$$

Important points:

1. Gravitational force and spring force are always non impulsive.
2. An impulsive force can only be balanced by another impulsive force.

Coefficient of restitution (  $e$  )

$$e = \frac{\text{Impulse of reformation}}{\text{Impulse of deformation}} = \frac{F_r dt}{F_d dt} = \frac{\text{Velocity of separation along line of impact}}{\text{velocity of approach along line of impact}}$$

a)  $e = 1$                       impulse of reformation = impulse of deformation

velocity of separation = velocity of approach

kinetic energy may be conserved

elastic collision

b)  $e = 0$                       impulse of reformation = 0

velocity of separation = 0

kinetic energy is not conserved

perfectly inelastic collision

c)  $0 < e < 1$                       impulse of reformation < impulse of deformation

velocity of separation < velocity of approach

kinetic energy is not conserved

inelastic collision

➤ Variable mass system:

If a mass is added or ejected from a system, at rate  $\mu$  kg/s and relative velocity  $v_{rel}$  (w.r.t. the system), then the force exerted by this mass on the system has magnitude  $\mu|v_{rel}|$

Thrust force ( $F_t$ )

$$F_t = v_{rel} \frac{dm}{dt}$$

➤ Rocket propulsion:

If gravity is ignored and initial velocity of the rocket  $u = 0$

$$v = v_r \ln$$

### Rigid body dynamics

1. Rigid body:

If the above body is rigid

$$V_A \cos \theta_1 = V_B \cos \theta_2$$

$V_{BA}$  = relative velocity of point B with respect to point A.

2. Moment of inertia (I)

Definition: Moment of inertia is defined as the capability of system to oppose the change produced in the rotational motion of a body.

Moment of inertia is a scalar positive quantity.

$$I = mr_1^2 + m_2r_2^2 + \dots$$

$$= I_1 + I_2 + I_3 + \dots$$

Si units of moment of inertia is  $\text{Kgm}^2$ .



➤ Moment of inertia of:

$$2.1 \text{ A single particle : } I = mr^2$$

Where  $m$  = mass of the particle

$r$  = perpendicular distance of the particle from the axis about which moment of inertia is to be calculated

2.2 For many particles (system of particles):

$$I = \sum_{i=1}^n m_i r_i^2$$

2.3 For a continuous object:

$$I = \int dm r^2$$

Where  $dm$  = mass of a small element

$r$  = perpendicular distance of the particle from the axis

2.4 For a larger object:

$$I = \int dI_{\text{element}}$$

Where  $dI$  = moment of inertia of a small element

Two important theorems on moment of inertia:

3.1 Perpendicular axis theorem

[only applicable to plane lamina (that means for 2-D objects only)]

$$I_z = I_x + I_y \text{ (when object is in x-y plane)}$$

3.2 Parallel axis theorem

(applicable to any type of object):

$$I_{AB} = I_{\text{cm}} + Md^2$$

List of some useful formula:

Object moment of inertia

Solid sphere  $\frac{2}{5}MR^2$  (Uniform)

Hollow sphere  $\frac{2}{3}MR^2$  (Uniform)

Ring  $\frac{MR^2}{2}$  (Uniform)

Disc  $MR^2$  (Uniform)

Hollow cylinder  $\frac{MR^2}{2}$  (Uniform)

Solid cylinder  $I_{AB} = I_{CD} = I_{DF} = \frac{Ma^2}{12}$  (Uniform)

Square plate  $\frac{Ma^2}{6}$  (Uniform)

Rectangular plate  $I = \frac{M(a^2 + b^2)}{12}$  (Uniform)

Cuboid  $\frac{M(a^2 + b^2)}{12}$  (Uniform)

3. Radius of gyration:

$$I = MK^2$$

4. Torque:

$$\tau = rF$$

5.5 relation between ' $\tau$ ' & ' $\alpha$ ' (for hinged object or pure rotation)

$$\tau_{ext} )_{Hinge} = I_{Hinge} \alpha$$

Where  $\tau_{\text{ext}})_{\text{Hinge}}$  is net external torque acting on the body about hinge point

$I_{\text{Hinge}}$  is moment of inertia of body about Hinge point

$$F_{1t} = M_1 a_{1t} = M_1 r_1 \alpha$$

$$F_{2t} = M_2 a_{2t} = M_2 r_2 \alpha$$

$$\tau_{\text{resultant}} = F_{1t} r_1 + F_{2t} r_2 + \dots$$

$$= M_1 \alpha r_1^2 + M_2 \alpha r_2^2 + \dots$$

$$\tau_{\text{resultant}})_{\text{external}} = I \alpha$$

Rotational kinetic energy =  $\frac{1}{2} I \omega^2$

$$P = M v_{\text{CM}} \quad F_{\text{external}} = M a_{\text{CM}}$$

Net external force acting on the body has two parts tangential and centripetal.

$$F_C = m a_C = m \frac{v^2}{r_{\text{cm}}} = m \omega^2 r_{\text{cm}} \quad F_t = m a_t = m \alpha r_{\text{cm}}$$

- Rotational Equilibrium
- For translational equilibrium

$$\Sigma F_x = 0 \dots \dots \dots \text{(i)}$$

$$\text{And } \Sigma F_y = 0 \dots \dots \dots \text{(ii)}$$

The condition of rotational equilibrium is  $\Sigma T_z = 0$

## 5. Angular momentum

### 7.1 angular momentum of a particle about a point

$$L = r P \quad L = r p \sin \theta$$

$$|L| = r_{\perp} P$$

$$|L| = P_{\perp} r$$

### 7.3 Angular momentum of a rigid body rotating about fixed axis:

$$L_H = I_H \omega$$

$L_H$  = angular momentum of object about axis H.

$I_H$  = momentum of inertia of rigid object about axis H.

$\omega$  = angular velocity of the object

### 7.4 Conservation of angular momentum

Angular momentum of a particle or system remains constant if  $\tau_{ext} = 0$  about that point or axis of rotation.

### 7.5 relation between torque and angular momentum

$$\tau = \frac{dL}{dt}$$

### 7.6 impulse of torque:

$$\tau dt = \Delta J$$

$\Delta J$  d change in angular momentum

For rigid body, the distance between the particles remains unchanged during its motion i.e.  $r_{P/Q} = \text{constant}$

➤ For velocities  $V_p = \sqrt{V_Q^2 + (\omega r)^2 + 2V_Q \omega r \cos \theta}$

➤ For acceleration:

$\theta, \omega, \alpha$  are same about every point of the body (or any other point outside which is rigidly attached to the body).

➤ Dynamics:

$$\tau_{cm} = I_{cm} \alpha, F_{ext} = Ma_{cm} \quad P_{system} = Mv_{cm}$$

$$\text{Total K.E.} = \frac{1}{2} Mv_{cm}^2 + \frac{1}{2} I_{cm} \omega^2$$

Angular momentum axis AB=L about C.M. +  $\vec{L}$  of C.M. about AB

$$L_{AB} = I_{cm} \omega + r_{cm} Mv_{cm}$$

### Simple harmonic motion

➤ S.H.M.  $F = kx$

➤ General equation of S.H.M. is  $x = A \sin(\omega t + \phi)$ ;  $(\omega t + \phi)$  is phase of the motion and  $\phi$  is initial phase of the motion.

➤ Angular frequency:  $\omega = \frac{2\pi}{T} = 2\pi f$

➤ Time period:  $T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{m}{k}}$

➤ Speed:  $v = \omega \sqrt{A^2 - x^2}$

➤ Acceleration:  $a = -\omega^2 x$

➤ Kinetic energy:  $\frac{1}{2} mv^2 = \frac{1}{2} m\omega^2 (A^2 - x^2) = \frac{1}{2} k(A^2 - x^2)$

➤ Potential energy (PE):  $\frac{1}{2} kx^2$

➤ Total mechanical energy (TME) =  $K.E. + P.E. = \frac{1}{2} k(A^2 - x^2) + \frac{1}{2} kx^2 = \frac{1}{2} kA^2$

(which is constant)

➤ Spring-Mass system

$$(i) \quad T = 2\pi\sqrt{\frac{m}{k}}$$

$$(ii) \quad T = 2\pi\sqrt{\frac{\mu}{K}}, \text{ where } \mu = \frac{m_1 m_2}{(m_1 + m_2)} \text{ known as reduced mass.}$$

➤ Superposition of SHM s along the same direction:

$$x_1 A \sin \omega t \text{ \& } x_2 = A_2 \sin(\omega t + \phi)$$

If equation of resultant SHM is taken as  $x = A \sin(\omega t + \phi)$

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1 A_2 \cos \theta} \quad \& \quad \tan \phi = \frac{A_2 \sin \theta}{A_1 + A_2 \cos \theta}$$

### String waves

➤ General equation of wave motion

$$\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}$$

$$y(x, t) = f\left(t \pm \frac{x}{v}\right)$$

Where,  $y(x, t)$  should be finite everywhere.

$ft + \frac{x}{v}$  represents wave travelling in –ve x-axis.

$ft - \frac{x}{v}$  represents wave travelling in +ve axis.

$$y = A \sin(\omega t \pm kx + \phi)$$

➤ Terms related to wave motion ( for 1-d progressive sine wave)

e) Wave number (or propagation constant) (k)

$$k = 2\pi / \lambda = \frac{\omega}{v} (\text{radm}^{-1})$$

f) Phase of wave: the argument of harmonic function ( $\omega t \pm kx + \phi$ ) is called phase of the wave.

Phase difference ( $\Delta\phi$ ): difference in phases of two particles at any time  $t$ .

$$\Delta\phi = \frac{2\pi}{\lambda} \Delta x \quad \text{Also. } \Delta\phi = \frac{2\pi}{T} \cdot \Delta t$$

Speed of transverse wave along a string/wire

$$v = \sqrt{\frac{T}{\mu}} \quad \text{where } T = \text{tension}$$

$\mu =$  mass per unit length

Power transmitted along the spring by a sine wave

$$\text{Average power } \langle P \rangle = 2\pi^2 f^2 A^2 \mu v$$

$$\text{Intensity } I = \frac{\langle P \rangle}{S} = 2\pi^2 f^2 A^2 \rho v$$

Reflection and refraction of waves

$$y = A \sin(\omega t - k_1 x)$$

$$y_t = A_t \sin(\omega t - k_2 x) \quad \text{if incident from rarer to denser medium } (V_2 < V_1)$$

$$y_r = -A_r \sin(\omega t + k_1 x)$$

$$y_t = A_t \sin(\omega t - k_2 x) \quad \text{If incident from denser to rare medium } (V_2 > V_1)$$

$$y_r = -A_r \sin(\omega t + k_1 x)$$

c) Amplitude of reflected and transmitted waves

$$A_r = \frac{|k_1 - k_2|}{k_1 + k_2} A_i \text{ \& } A_t = \frac{2k_1}{k_1 + k_2} A_i$$

Standing/ stationary waves:-

$$y_1 = A \sin(\omega t - kx + \theta_1)$$

(b)  $y_2 = A \sin(\omega t - kx + \theta_2)$

$$y_1 + y_2 = 2A \cos kx + \frac{\theta_2 - \theta_1}{2} \sin \omega t + \frac{\theta_1 + \theta_2}{2}$$

The quantity  $2A \cos kx + \frac{\theta_2 - \theta_1}{2}$  represents resultant amplitude at x. At some position resultant amplitude is zero these are called nodes. At some positions resultant amplitude is 2A, these are called antinodes.

d) Distance between successive nodes or antinodes =  $\frac{\lambda}{2}$

e) Distance between successive nodes and antinodes =  $\frac{\lambda}{4}$

f) All the particles in same segment (portion between 2 successive nodes) vibrate in the same phase.

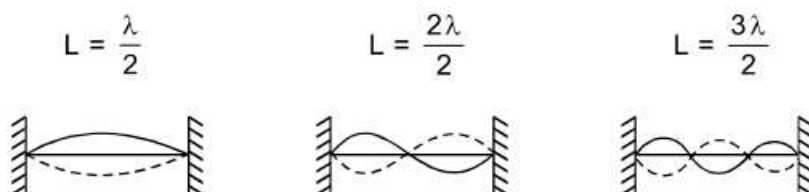
g) The particles in 2 consecutive segments vibrate in opposite phase.

h) Since nodes are permanently at rest so energy cannot be transmitted across these.

Vibrations of strings (standing wave)

a) Fixed at both ends:

Fixed ends will be nodes. So waves for which





are possible giving

$$L = \frac{n\lambda}{2} \text{ or } \lambda = \frac{2L}{n} \text{ where } n=1,2,3,\dots$$

$$\text{As } v = \sqrt{\frac{T}{\mu}} \quad f_n = \frac{n}{2L} \sqrt{\frac{T}{\mu}}, n = \text{no. of loops}$$

b) String free at one end:

$$1. \text{ For fundamental mode } L = \frac{\lambda}{4} \text{ or } \lambda = 4L$$

$$\text{First overtone } L = \frac{3\lambda}{4}, \text{ hence } \lambda = \frac{4L}{3}$$

$$\text{So, } f_1 = \frac{3}{4L} \sqrt{\frac{T}{\mu}} \text{ (first overtone)}$$

$$\text{Second overtone } f_2 = \frac{5}{4L} \sqrt{\frac{T}{\mu}}$$

$$\text{So, } f_n = \frac{n + \frac{1}{2}}{2L} \sqrt{\frac{T}{\mu}} = \frac{(2n+1)}{4L} \sqrt{\frac{T}{\mu}}$$

### Heat & thermodynamics

$$\text{Total translational K.E. of gas} = \frac{1}{2} M \langle V^2 \rangle = \frac{3}{2} PV = \frac{3}{2} nRT$$

$$\langle V^2 \rangle = \frac{3P}{\rho} \quad V_{rms} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3RT}{M_{mol}}} = \sqrt{\frac{3KT}{m}}$$

Important points:  $V_{rms} \sqrt{T} \quad \bar{v} = \sqrt{\frac{8KT}{\pi m}} = 1.59 \sqrt{\frac{KT}{m}} \quad V_{rms} = 1.73 \sqrt{\frac{KT}{m}}$

Most probable speed  $V_p = \sqrt{\frac{2KT}{m}} = 1.41 \sqrt{\frac{KT}{m}}$   
 $\therefore V_{rms} > \bar{v} > V_{mp}$

Degree of freedom:

Mono atomic  $f = 3$

Diatomic  $f = 5$

Polyatomic  $f = 6$

Maxwell's law of equipartition of energy:

Total K.E. of the molecule =  $\frac{1}{2} fKT$

For an ideal gas:

Internal energy  $U = \frac{f}{2} nRT$

Work done in isothermal process:  $W = \left[ 2.303nRT \log_{10} \frac{V_f}{V_i} \right]$

➤ Internal energy in isothermal process:

$$\Delta U = 0$$

➤ Work done in isochoric process:

$$dW = 0$$

➤ Change in int. energy in isochoric process:

$$\Delta U = n \frac{f}{2} R \Delta T = \text{heat given}$$

➤ Isobaric process:

$$\text{Work done } \Delta W = nRT(T_f - T_i)$$

$$\text{Change in int. energy } \Delta U = nC_v\Delta T$$

$$\text{Heat given } \Delta Q = \Delta U + \Delta W$$

$$\text{Specific heat: } C_v = \frac{f}{2}R \quad C_p = \frac{f}{2}R + R$$

Molar heat capacity of ideal gas in terms of R:

i) For monoatomic gas:  $\frac{C_p}{C_v} = 1.67$

ii) For diatomic gas:  $\frac{C_p}{C_v} = 1.4$

iii) for triatomic gas:  $\frac{C_p}{C_v} = 1.33$

➤ In general:  $\gamma = \frac{C_p}{C_v} = 1 + \frac{2}{f}$

➤ Mayer's eq.  $C_p - C_v = R$  for ideal gas only

➤ Adiabatic process:

➤ Work done  $\Delta W = \frac{nRT(T_i - T_f)}{\gamma - 1}$

- In cyclic process:
- $\Delta Q = \Delta W$
- In a mixture of non-reacting gases:

$$\text{Mol. wt.} = \frac{n_1 M_1 + n_2 M_2}{n_1 + n_2}$$

$$C_v = \frac{n_1 C_{v1} + n_2 C_{v2}}{n_1 + n_2}$$

$$\gamma = \frac{C_p(\text{mix})}{C_v(\text{mix})} = \frac{n_1 C_{p1} + n_2 C_{p2} + \dots}{n_1 C_{v1} + n_2 C_{v2} + \dots}$$

- Calorimetry and thermal expansion

Types of thermometers:

a) Liquid thermometer:  $T = \frac{l - l_0}{l_{100} - l_0} 100$

b) Gas thermometer:

Constant volume:  $T = \frac{P - P_0}{P_{100} - P_0} 100$ ;  $P = P_0 + \rho gh$

Constant pressure:  $T = \frac{V}{V - V_0} T_0$

c) Electrical resistance thermometer:

$$T = \frac{R_t - R_0}{R_{100} - R_0} 100$$

Thermal expansion:

a) Linear:

$$\alpha = \frac{\Delta L}{L_0 \Delta T} \text{ or } L = L_0 (1 + \alpha \Delta T)$$

b) Area/superficial:

$$\beta = \frac{\Delta A}{A_0 \Delta T} \text{ or } A = A_0 (1 + \beta \Delta T)$$

c) Volume/cubical:

$$\gamma = \frac{\Delta V}{V_0 \Delta T} \text{ or } V = V_0 (1 + \gamma \Delta T)$$

$$\alpha = \frac{\beta}{2} = \frac{\gamma}{3}$$

➤ Thermal stress of a material:

$$\frac{F}{A} = Y \frac{\Delta \ell}{\ell}$$

➤ Energy stored per unit volume:

$$E = \frac{1}{2} K (\Delta L)^2 \text{ or } E = \frac{1}{2} \frac{AY}{L} (\Delta L)^2$$

➤ Variation of time period of pendulum clocks:

$$\Delta T = \frac{1}{2} \Delta \theta T$$

$T < T$  -clock fast: time gain

$T > T$  -clock slow: time loss

➤ Calorimetry:

$$\text{Specific heat } S = \frac{Q}{m \Delta T}$$

Molar specific heat  $C = \frac{\Delta Q}{N \cdot \Delta T}$

Water equivalent  $= m_w S_w$

➤ Heat transfer

Thermal conduction:  $\frac{dQ}{dt} = KA \frac{dT}{dx}$

Thermal resistance:  $R = \frac{\ell}{KA}$

Series and parallel combination of rod:

I) Series:  $\frac{\ell_{eq}}{K_{eq}} = \frac{\ell_1}{K_1} + \frac{\ell_2}{K_2} + \dots$  (when  $A_1 = A_2 = A_3 = \dots$ )

II) Parallel:  $K_{eq} A_{eq} = K_1 A_1 + K_2 A_2 + \dots$  (when  $\ell_1 = \ell_2 = \ell_3 = \dots$ )

for absorption, reflection and transmission

$$r + t + a = 1$$

➤ Emissive power:  $E = \frac{\Delta U}{\Delta A \Delta t}$

➤ Spectral emissive power:  $E_\lambda = \frac{dE}{d\lambda}$

➤ Emissivity:  $e = \frac{\text{E of a body at T temp.}}{\text{E of a body at T temp.}}$

➤ Kirchhoff's law:  $\frac{E(\text{body})}{a(\text{body})} = E(\text{black body})$

➤ Wein's displacement law:  $\lambda_m T = b$ .

$$b = 0.282 \text{ cm-k}$$

➤ Stefan Boltzmann law:

$$u = \sigma T^4 \text{ s} = 5.67 \times 10^8 \text{ W / m}^2 \text{ k}^4$$

$$\Delta u = u - u_0 = e\sigma A(T^4 - T_0^4)$$

➤ Newton's law of cooling:  $\frac{d\theta}{dt} = k(\theta - \theta_0); \theta = \theta_0 + (\theta - \theta_0)e^{-kt}$

## Electrostatics

Column force between 2 point charges  $F = \frac{1}{4\pi\epsilon_0\epsilon_r} \frac{q_1q_2}{|r|^3} r = \frac{1}{4\pi\epsilon_0\epsilon_r} \frac{q_1q_2}{|r|^2} r$

- Electric field intensity at any point is the force experienced by unit positive charge given by  $E = F/q_0$

Electric force on a charge 'q' at position of electric field intensity E produced by some source charges is  $F = qE$

Electric potential

If  $(W_p)_{ext}$  is the work required in moving a point charge q from Infinity to a point P, the electric potential of the point P is

$$V_p = \frac{(W_p)_{ext}}{q} \quad \text{acc}=0$$

Potential difference between 2 points A and B is

$$V_A - V_B$$

Formula of E and potential V

I) Point charge  $E = \frac{Kq}{|r|^2} r = \frac{kq}{r^3} r, V = \frac{Kq}{r}$

II) Infinitely long line charge  $\frac{\lambda}{2\pi\epsilon_0 r} r = \frac{2k\lambda r}{r}$

V = not defined  $V_A - V_B = 2K\lambda \ln(r_B / r_A)$

III) Infinite non conducting thin sheet  $\frac{\sigma}{2\epsilon_0} n$

V = not defined  $v_B - v_A = -\frac{\sigma}{2\epsilon_0}(r_B - r_A)$

IV) Uniformly charged ring

$$E_{axis} = \frac{KQx}{(R^2 + x^2)^{3/2}}, E_{centre} = 0$$

$$V_{axis} = \frac{KQ}{\sqrt{R^2 + x^2}}, V_{centre} = \frac{KQ}{R}$$

x is the distance from centre along axis.

V) Infinitely large charged conducting sheet  $\frac{\sigma}{\epsilon_0} n$

V = not defined,  $v_B - v_A = -\frac{\sigma}{\epsilon_0}(r_B - r_A)$

VI) Uniformly charged hollow conducting/non conducting/solid conducting sphere



(a) For  $E = \frac{kQ}{|r|^2}$  for  $r > R$ ,  $V = \frac{kQ}{r}$

(b)  $E = 0$  for  $r < R$ ,  $V = \frac{kQ}{R}$

VII) Uniformly charged solid non conducting sphere (insulating material)

(a)  $E = \frac{kQ}{|r|^2}$  for  $r > R$ ,  $V = \frac{kQ}{r}$

(b)  $E = \frac{KQr}{R^3} = \frac{\rho r}{3\epsilon_0}$  for  $r < R$ ,  $V = \frac{\rho}{6\epsilon_0}(3R^2 - r^2)$

VIII) Thin uniformly charged disc (surface charge density is  $\sigma$ )

$$E_{axis} = \frac{\sigma}{2\epsilon_0} \left( 1 - \frac{x}{\sqrt{R^2 + x^2}} \right)$$

$$V_{axis} = \frac{\sigma}{2\epsilon_0} \left( \sqrt{R^2 + x^2} - x \right)$$

Work done by external agent in taking a charge  $q$  from A to B is

$$(W_{ext})_{AB} = q(V_B - V_A) \text{ or } (W_B)_{AB} = q(V_A - V_B)$$

The electrostatic potential energy of point charge

$$U = qV$$

$U =$  PE of the system

$$\frac{U_1 + U_2 + \dots}{2} = (U_{12} + U_{13} + \dots + U_{1n}) + (U_{23} + U_{24} + \dots + U_{2n}) + (U_{34} + U_{35} + \dots + U_{3n})$$

$$\text{Energy density} = \frac{1}{2} \epsilon E^2$$

$$\text{Self energy of a uniformly charged shell } U_{self} = \frac{KQ^2}{2R}$$

Self energy of a uniformly charged solid non conducting sphere  $U_{self} = \frac{3KQ^2}{5R}$

Electric field intensity due to dipole

I) On the axis  $E = \frac{2KP}{r^3}$

II) On the Equatorial position  $E = \frac{KP}{r^3}$

III) Total electric field at journal point  $O(r, \theta)$

$$E_{res} = \frac{KP}{r^3} \sqrt{1 + 3\cos^2 \theta}$$

Potential energy of an electric dipole in external electric field

$$U = -\vec{E} \cdot \vec{p}$$

Electric dipole in uniform electric field

Torque  $\vec{\tau} = \vec{E} \times \vec{p}; \vec{F} = 0$

Electric dipole in non uniform electric field

$$\vec{\tau} = \vec{E} \times \vec{p}; U = -pE, \text{ net force } |F| = \left| p \frac{dE}{dr} \right|$$

Electric potential due to dipole at general point

$$V = \frac{P \theta}{4\pi e_0^2} = \frac{\cdot}{4\pi e_0^3}$$

The electric flux over the whole area is given by

$$\phi_F = E_s \cdot \vec{dS} = \int E_n dS$$

Flux using Gauss's law, flux through a closed surface

$$\phi_F = \oint E \cdot \vec{dS} = \frac{q_{in}}{e_0}$$

Electric field intensity near the conducting surface

$$= \frac{\sigma}{e_0} n$$

Electric pressure: electric pressure at the surface of a conductor is given by formula

$$P = \frac{\sigma^2}{2e_0}$$

Where  $\sigma$  is the local surface charge density

Potential difference between points A and B

$$V_B - V_A = \int_A^B E \cdot dr$$

$$E = -i \frac{\partial V}{\partial x} + j \frac{\partial V}{\partial y} + k \frac{\partial V}{\partial z} = -i \frac{\partial V}{\partial x} + j \frac{\partial V}{\partial y} + k \frac{\partial V}{\partial z}$$

$$= -\text{grad } V$$

## Current electricity

1. Electric current  $I_{av} = \frac{\Delta q}{\Delta t}$  and instantaneous current

$$i = \lim_{\Delta t \rightarrow 0} \frac{\Delta q}{\Delta t} = \frac{dq}{dt}$$

## 2. Electric current in a conductor

$$I = nAeV$$

$$V_d = \frac{\lambda}{\tau},$$

$$V_d = \frac{\frac{1}{2} \frac{eE}{m} \tau^2}{\tau} = \frac{1}{2} \frac{eE}{m} \tau,$$

$$I = neAV_d$$

## 3. Current density $\vec{J} = \frac{dl}{ds} \vec{n}$

## 4. Electrical resistance $I = neAV_d = neA \frac{eE}{2m} \tau = \frac{ne^2 \tau}{2m} AE$

$\rho$  is called resistivity (it is also called specific resistance) and  $\rho = \frac{2m}{ne^2 \tau} = \frac{1}{\sigma}$ ,  $\sigma$  is called conductivity.

Therefore current in conductors is proportional to potential difference applied across its ends. This is ohm's law.

Units: R ohm( $\Omega$ ),  $\rho$  ohm-meter( $\Omega\text{-m}$ ) also called Siemens,  $\sigma \Omega^{-1}\text{m}^{-1}$ .

Dependence of resistance on temperature:

$$R = R_0 (1 + \alpha\theta)$$

Electric current in resistance

$$I = \frac{V_2 - V_1}{R}$$

## 5. Electrical power

$$P = VI$$

Energy =  $pdt$

$$P = I^2 R = VI = \frac{V^2}{R}$$

$$H = VIt = I^2 Rt = \frac{V^2}{R} t$$

$$H = I^2 RT \text{ Joule} = \frac{I^2 RT}{4.2} \text{ calorie}$$

## 9. Kirchhoff's law

### 9.1 Kirchhoff's current law (junction law)

$$\sum I_{in} = \sum I_{out}$$

### 9.2 Kirchhoff's voltage law (loop law)

$$\sum IR + \sum EMF = 0$$

## 11. Combination of resistances:

Resistances in series:

$R = R_1 + R_2 + R_3 + \dots + R_n$  (this means  $R_{eq}$  is greater than any resistor) and

$$V = V_1 + V_2 + V_3 + \dots + V_n$$

$$V_1 = \frac{R_1}{R_1 + R_2 + R_3 + \dots + R_n} V; V_2 = \frac{R_2}{R_1 + R_2 + R_3 + \dots + R_n} V;$$

## 2. Resistances in parallel:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

## 12. Wheatstone network: (4 terminal network)

When current through the galvanometer is zero (null point or balance point)  $P/Q = R/S$ , then  $PS = QR$

## 13. Grouping of cells

### 13.1 Cells in series:

Equivalent EMF  $E_{eq} = E_1 + E_2 + \dots + E_N$  [write EMF's with polarity]

Equivalent internal resistance  $r_{eq} = r_1 + r_2 + r_3 + r_4 + \dots + r_n$

### 13.2 Cells in parallel:

$$E_{eq} = \frac{\frac{e_1}{r_1} + \frac{e_2}{r_2} + \dots + \frac{e_n}{r_n}}{\frac{1}{r_1} + \frac{1}{r_2} + \dots + \frac{1}{r_n}} \text{ [Use emf with polarity]}$$

$$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} + \dots + \frac{1}{r_n}$$

### 14. Ammeter

A shunt (small resistance) is connected in parallel with galvanometer to convert it into ammeter. An ideal ammeter has zero resistance.

Ammeter is represented as follows –

If maximum value of current to be measured by ammeter, is  $I$  then

$$I_G \cdot R_G = (I - I_G) S$$

$$S = \frac{I_G \cdot R_G}{I - I_G}, S = \frac{I_G R_G}{I} \text{ when } I \gg I_G$$

Where,  $I$  = maximum current that can be measured using the given ammeter.

### 15. Voltmeter

A high resistance is put in series with galvanometer. It is used to measure potential difference across a resistor in a circuit.

For maximum potential difference

$$V = I_G \cdot R_S + I_G R_G$$

$$R_S = \frac{V}{I_G} R_G \text{ if } R_G \ll R_S$$

$$R_S = \frac{V}{I_G}$$

## 16. Potentiometer

$$I = \frac{e}{r + R}$$

$$V_A - V_B = \frac{e}{R + r} \cdot R$$

Potential gradient (x)      Potential difference per unit length of wire

$$x = \frac{V_A - V_B}{L} = \frac{e}{R + r} \cdot \frac{R}{L}$$

➤ Application of potentiometer

a) To find emf of unknown cell and compare emf of two cells

In case I,

In figure (1) is joint to (2) then balance length =  $\ell_1$

$$e_1 = x \ell_1 \dots (1)$$

In case II,

In figure (3) is joint to (2) then balance length =  $\ell_2$

$$e_2 = x \ell_2$$

$$\frac{e_1}{e_2} = \frac{\ell_1}{\ell_2}$$

If any one of  $e_1$  or  $e_2$  is known the other can be found. If  $x$  is known then both  $e_1$  and  $e_2$  can be found.

b) To find current if resistance is known

$$V_A - V_C = x\ell_1$$

$$IR_1 = x\ell_1$$

$$I = \frac{x\ell_1}{R_1}$$

Similarly, we can find the value of  $R_2$  also.

Potentiometer is ideal voltmeter because it does not draw any current from circuit, at the balance point.

c) To find the internal resistance of cell

1<sup>st</sup> arrangement

$$e = x\ell_1$$

2<sup>nd</sup> arrangement

$$IR = x\ell_2$$

$$I = \frac{x\ell_2}{R}, \text{ also } I = \frac{e'}{r' + R}$$

$$\therefore \frac{e'}{r' + R} = \frac{x\ell_2}{R} \quad \frac{x\ell_1}{r' + R} = \frac{x\ell_2}{R}$$

$$r = \frac{\ell_1 - \ell_2}{\ell_2} R$$

d) Ammeter and voltmeter can be graduated by potentiometer.

e) Ammeter and voltmeter can be calibrated by potentiometer.

17. Metre bridge (use to measure unknown resistance)

If  $AB = l$  cm, then  $BC = (100 - l)$  cm



Resistance of the wire between A and B,  $R = \sigma l$

[  $\sigma$  specific resistance and cross-sectional area A are same for whole of the wire]

$$\text{Or } R = \sigma l$$

Where  $\sigma$  is resistance per cm of wire.

If P is the resistance of wire between A and B then

$$P = \sigma l \quad \text{P} = \sigma(l)$$

Similarly if Q is the resistance of the wire between B and C, then

$$Q = \sigma(100 - l) \quad \dots(2)$$

$$\text{Dividing (1) by (2)} \quad \frac{P}{Q} = \frac{l}{100 - l}$$

Applying the condition for balanced Wheatstone bridge, we get  $RQ = PX$

$$\therefore x = R \frac{Q}{P} \text{ or } X = \frac{100 - l}{l} R$$

Since R and  $l$  Are known therefore the value of X can be calculated.

### Capacitance

$$1. \text{ I) } q = CV \quad q = CV$$

q: charge on positive plate of the capacitor

C: capacitance of capacitor

V: potential difference between positive and negative plates

III) Representation of capacity:

$$\text{IV) Energy stored in the capacitor: } U = \frac{1}{2} CV^2 = \frac{Q^2}{2C} = \frac{QV}{2}$$

$$V) \quad \text{Energy density} = \frac{1}{2} \epsilon_0 \epsilon_r E^2 = \frac{1}{2} \epsilon_0 K E^2$$

$\epsilon_r$  = relative permittivity of the medium       $K = \epsilon_r$  ; Dielectric constant

VI) Types of capacitors:

a) Parallel plate capacitor

$$C = \frac{\epsilon_0 \epsilon_r A}{d} = K \frac{\epsilon_0 A}{d}$$

A: area of plates

d: distance between the plates ( $\ll$  size of plate)

b) Spherical capacitor:

Capacitors of an isolated spherical conductor (hollow or solid)

$$C = 4\pi\epsilon_0\epsilon_r R$$

R = radius of spherical conductor

Capacitance of spherical conductor

$$C = 4\pi\epsilon_0 \frac{ab}{(b-a)}$$

$$C = \frac{4\pi\epsilon_0 K_2 ab}{(b-a)}$$

c) Cylindrical capacitor:  $l \gg \{a, b\}$

$$\text{Capacitance per unit length} = \frac{2\pi\epsilon_0}{\ln(b/a)} F/m$$

VII) Capacitance of capacitor depends on

a) Area of plates

b) Distance between the plates

c) Dielectric medium between the plates

VIII) Electric field intensity between the plates of capacitor

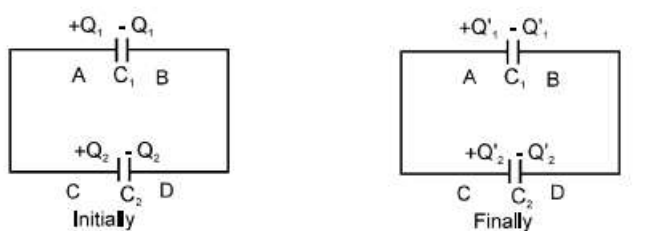
$$E = \frac{\sigma}{\epsilon_0} = \frac{V}{d}$$

$\sigma$ : surface charge density

IX) Force experienced by any plate of capacitor:  $F = \frac{q^2}{2A\epsilon_0}$

2. Distribution of charges on connecting 2 charged capacitors:

❖ When 2 capacitors are  $C_1$  and  $C_2$  are connected as shown in the figure



a) Common potential:

$$V = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2} = \frac{\text{Total charge}}{\text{Total Capacitance}}$$

b)  $Q'_1 = C_1 V = \frac{C_1}{C_1 + C_2} (Q_1 + Q_2)$

$$Q'_2 = C_2 V = \frac{C_2}{C_1 + C_2} (Q_1 + Q_2)$$

c) Heat loss during redistribution:  $\Delta H = U.U_f = \frac{1}{2} \frac{C_1 C_2}{C_1 + C_2} (V_1 - V_2)^2$

3. Combination of capacitor:

I) Series combination

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$V_1 : V_2 : V_3 = \frac{1}{C_1} : \frac{1}{C_2} : \frac{1}{C_3}$$

## II) Parallel combination

$$C_{eq} = C_1 + C_2 + C_3$$

$$Q_1 : Q_2 : Q_3 = C_1 : C_2 : C_3$$

### 4. Charging and discharging of a capacitor:

#### I) Charging of a capacitor (capacitor initially uncharged):

$$q = q_0 (1 - e^{-t/\tau})$$

$q_0$  = charge on the capacitor at steady state

$$q_0 = CV$$

$\tau$  : time constant =  $CR_{eq}$

$$I = \frac{q_0}{\tau} e^{-t/\tau} = \frac{V}{R} e^{-t/\tau}$$

#### II) Discharging of capacitor:

$$q = q_0 e^{-t/\tau}$$

$q_0$  = initial charge on the capacitor

### 5. Capacitor with dielectric:

#### I) Capacitance in the presence of dielectric:

$$C = \frac{K \epsilon_0 A}{d} = KC_0$$

$C_0$  = capacitance in the absence of dielectric.

$$(II) \quad E_n = E E_{n0} = \frac{\sigma}{\epsilon_0} \frac{\sigma_b}{\epsilon_0} = \frac{\sigma}{K\epsilon_0} = \frac{V}{d}$$

$E: \frac{\sigma}{\epsilon_0}$  electric field in the absence of dielectric.

$E_{n0}$ : Induced (bound) charge density.

$$(III) \quad \sigma_0 = \sigma \left( 1 - \frac{1}{K} \right)$$

## 6. Force on dielectric:

I) When battery is connected

$$F = \frac{\epsilon_0 b (k-1) V^2}{2d}$$

II) When battery is not connected

$$F = \frac{Q^2}{2C^2} \frac{dC}{dx}$$

\*Force on the dielectric will be zero when the dielectric is fully inside.

## Alternating current

### 1. AC and DC current

A current that changes its direction periodically is called alternating current (AC). If a current maintains direction constant it is for direct current (DC).

### 2. Root mean square value:

Root mean square value of a function,

$t_1$  to  $t_2$ , is defined as  $f_{rms} = \sqrt{\frac{\int_{t_1}^{t_2} f^2 dt}{t_2 - t_1}}$

### 3. Power consumed or supplied in an AC circuit:

Average power consumed in a cycle

$$= \frac{\int_0^{2\pi} P dt}{2\pi} = \frac{1}{2} V_m I_m \cos \phi$$

$$= \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cos \phi = V_{rms} I_{rms} \cos \phi$$

Here  $\cos \phi$  is called power factor.

### 4. Some definitions:

The factor  $\cos \phi$  is called power factor.

Impedance  $Z$  is defined as  $Z = \frac{V_m}{I_m} = \frac{V_{rms}}{I_{rms}}$

$\omega L$  is called inductive reactance and is denoted by  $X_L$

$\frac{1}{\omega C}$  is called capacitive reactance and is denoted by  $X_C$

### 5. Purely resistive circuit:

$$I = \frac{v_s}{R} = \frac{V_m \sin \omega t}{R} = I_m \sin \omega t$$

$$I_m = \frac{V_m}{R}$$

$$I_{rms} = \frac{V_{rms}}{R}$$

$$\langle P \rangle = V_{rms} I_{rms} \cos \phi = \frac{V_{rms}^2}{R}$$

### 6. Purely capacitive circuit:

$$I = \frac{V_m}{\frac{1}{\omega C}} \cos \omega t$$

$$= \frac{V_m}{X_c} \cos \omega t = I_m \cos \omega t$$

$X_c = \frac{1}{\omega C}$  and is called capacitive reactance.

$I_c$  leads by  $V_c$  by  $\pi/2$  diagrammatically (phasor diagram) it is represented as

Since  $\phi = 90^\circ$ ,  $\langle P \rangle = V_{rms} I_{rms} \cos \phi = 0$

### Magnetic effect of current & magnetic force on charge/current

1. Magnetic field due to a moving point charge  $B = \frac{\mu_0}{4\pi} \frac{q(v \times r)}{r^3}$

2. Biot-savart's law  $d\vec{B} = \frac{\mu_0}{4\pi} \frac{d\vec{\ell} \times \vec{r}}{r^3}$

3. Magnetic field due to straight wire  $B = \frac{\mu_0}{4\pi} \frac{l}{r} (\sin \theta_1 + \sin \theta_2)$

4. Magnetic field due to infinite straight wire  $B = \frac{\mu_0}{2\pi} \frac{l}{r}$

➤ Magnetic field due to circular loop

I) At centre  $B = \frac{\mu_0 N I}{2r}$

II) At axis  $B = \frac{\mu_0}{2} \frac{N I R^2}{(R^2 + x^2)^{3/2}}$

➤ Magnetic field on the axis of the solenoid  $B = \frac{\mu_0 N I}{2} (\cos \theta_1 + \cos \theta_2)$

➤ Ampere's law  $\oint B \cdot d\ell = \mu_0 I$

$$B = 0, r < R$$

➤ Magnetic field due to long cylindrical shell  $= \frac{\mu_0 I}{2\pi r}, r > R$

➤ Magnetic force acting on a moving point charge

a.  $F = q(\mathbf{v} \times \mathbf{B})$

I)  $v \perp B$

$$r = \frac{mv}{qB}$$

$$T = \frac{2\pi m}{qB}$$

II)  $r = \frac{mv \sin \theta}{qB}$

$$T = \frac{2\pi m}{qB} \quad \text{pitch} = \frac{2\pi mv \cos \theta}{qB}$$

b.  $F = q[(\mathbf{v} \times \mathbf{B}) + \mathbf{E}]$

➤ Magnetic force acting on a current carrying wire  $F = I(\ell \times B)$

➤ Magnetic moment of a current carrying loop  $M = NIA$

➤ Torque acting on a loop  $\vec{\tau} = \vec{M} \times \vec{B}$

➤ Magnetic field due to a single pole  $B = \frac{\mu_0}{4\pi} \frac{m}{r^2}$

➤ Magnetic field on the axis of magnet  $B = \frac{\mu_0}{4\pi} \frac{2m}{r^3}$

➤ Magnetic field on the Equatorial axis of the magnet  $B = \frac{\mu_0}{4\pi} \frac{2m}{r^3}$

➤ Magnetic field at point P due to magnet  $B = \frac{\mu_0}{4\pi} \frac{m}{r^3} \sqrt{1 + 3\cos^2 \theta}$



## Electromagnetic induction

1. Magnetic flux is mathematically defined as  $\phi = B \cdot ds$
2. Faraday's law of electromagnetic induction  $E = \frac{d\phi}{dt}$
3. Lenz's law (conservation of energy principle)

According to this law, emf will be induced in such a way that it will oppose the cause which has produced it.

### Motional emf

4. Induced emf due to rotation

Emf induced in a conducting rod of length  $l$  rotating with angular speed  $\omega$  about its one end, in a uniform perpendicular magnetic field  $B$  is  $\frac{1}{2} B \omega l^2$ .

1. EMF induced in a rotating disc:

EMF between the centre and the edge of disc of radius  $r$  rotating in a magnetic field  $B = \frac{B \omega r^2}{2}$

5. Fixed loop in a varying magnetic field

If magnetic field changes with the rate  $\frac{dB}{dt}$ , electric field is generated whose average tangential value along a circle is given by  $E = -\frac{r}{2} \frac{dB}{dt}$

This electric field is non conservative in nature. The lines of force associated with this electric field are closed curves.

6. Self induction

$$E = \frac{\Delta(N\phi)}{\Delta t} = \frac{\Delta(LI)}{\Delta t} = \frac{L\Delta I}{\Delta t}$$

The instantaneous emf is given as  $E = -\frac{(N\phi)}{dt} = -\frac{(LI)}{dt} = \frac{LI}{dt}$

Self inductance of solenoid  $\mu_0 n^2 \pi r^2 \ell$

6.1 inductor

it is represented by

Electrical equivalence of loop

$$V_A - L \frac{dI}{dt} = V_B$$

Energy stored in an inductor  $= \frac{1}{2} LI^2$

### 7. Growth of current in series R L circuit

If a circuit consists of a cell, an inductor L and a resistor R and switch S, connected in series and the switch is closed at  $t = 0$ , the current in the circuit I

will increase as  $I = \frac{E}{R} \left( 1 - e^{-\frac{Rt}{L}} \right)$

The quantity  $L/R$  is called time constant of the circuit and is denoted by

The variation of current is with time is as shown

1. Final current in the circuit  $= E/R$ , which is independent of L.
2. After one time constant, current in the circuit  $= 63\%$  of the final current.
3. More time constant in the circuit implies lower rate of change of current.

### 8. Decay of current in the circuit containing resistor and inductor:

Let the initial current in a circuit containing inductor and resistor be  $I_0$ . Current

at a time  $t$  is given as  $I = I_0 e^{-\frac{Rt}{L}}$

Current after one time constant:  $I = I_0^{-1} = 0.37\%$  of initial current.

9. Mutual inductance is conduction of EMF in a coil (secondary) due to change in current in another coil (primary).

Or  $N \phi$  (in secondary) =  $MI$

The emf generated around the secondary due to the current flowing around the primary is directly proportional to the rate at which that current changes.

10. Equivalent self inductance:

$$L = \frac{V_A - V_B}{dl / dt}$$

1. Series combination:

$L = L_1 + L_2$  (neglecting mutual inductance)

$L = L_1 + L_2 + 2M$  (if coils are mutually coupled and they have winding in same direction)

$L = L_1 + L_2 - 2M$  (equals are mutually coupled and they have winding in opposite direction)

2. Parallel combination:

$1/L = 1/L_1 + 1/L_2$  (neglecting mutual inductance)

For two coils which are mutually coupled it has been found that  $M$

$$\sqrt{L_1 L_2} \text{ or } M = k \sqrt{L_1 L_2}$$

Work is called coupling constant and its value is less than or equal to one.

$\frac{E_0}{E_p} = \frac{N_s}{N_p} = \frac{I_p}{I_s}$ , where denotations have their usual meanings.

$N_s > N_p$   $E_s > E_p$  for step up transformer.

11.LC oscillations  $\omega^2 = \frac{1}{LC}$

## Geometrical optics

### 1. Reflection of light

c)  $i = r$

#### 1.3 Characteristics of image due to reflection by a plane mirror:

- Distance of object from mirror = distance of image from the mirror
- The line joining a point object and its image is normal to the reflecting surface
- The size of the image is same as that of the object
- For a real object the image is virtual and for a virtual object the image is real

#### 2. Relation between velocity of object and image:

From mirror property:  $x_m = -x_{om}$ ,  $y_m = -y_{om}$ , and  $z_m = -z_{om}$

Here  $x_m$  means x coordinate of image with respect to mirror.

Similarly, others have same meaning

Differentiating w.r.t. time, we get

$$v_{(m)x} = -v_{(om)x}; v_{(m)y} = -v_{(om)y}; v_{(m)z} = -v_{(om)z}$$

### 3. Spherical mirror

$$1 + 1 = \frac{2}{R} = 1 \dots \text{mirror formula}$$

x co ordinate of centre of curvature and focus of concave mirror are negative and those for convex mirror are positive.

In case of mirror since light rays reflect back in – x direction, therefore negative sign of v indicates real image and positive sign of v indicates virtual image.

b) Lateral magnification (or transverse magnification)  $m = \frac{v}{u}$

$M = -$ .

d) On differentiating (a) get  $\frac{dv}{du} = -\frac{2}{u}$

e) On differentiating (a) with respect to time we get  $\frac{dv}{dt} = -\frac{2}{u} \frac{du}{dt}$  where  $\frac{dv}{dt}$  Is the velocity of image along principle axis and  $\frac{du}{dt}$  Is the velocity of object along principle axis. Negative sign implies that the image in case of mirror always moves in the direction opposite to that of object. Is discussion is for velocity with respect to mirror and along the X axis.

f) Newton's formula:

$$XY = f^2$$

X & Y are the distances along the principal axis of the object and image respectively from the principal focus. Is formula can be used when the distances are mentioned asked from the focus.

g) Optical power of a mirror (in dioptries) =  $1/f$

f = focal length with sign and in meters

h) If object lying along the principal axis is mod of very small size, the longitudinal magnification =  $v_2 - v_1 / u_2 - u_1$  (it will always be involved)

#### 4. Refraction of light

##### 4.1 laws of refraction (at any refracting surface)

b)  $\sin i / \sin r = \text{constant}$  for any pair of media and for light of a given wave length. This is known as snell's law. More precisely,

##### 4.2 deviation of a ray due to refraction

Deviation ( $\delta$ ) of ray incident at  $i$  and refracted at  $r$  is given by  $\delta = |i - r|$

➤ Principal of reversibility of light rays

A ray travelling along the path of the reflected ray is reflected along the path of the incident ray. A refracted ray reversed through travel back along its path will get refracted along the path of the incident ray. Hush the incident and refracted rays are mutually reversible.

#### 5. Apparent depth and shift of submerged object

At near normal incidence (small angle of incidence  $i$ ) apparent depth ( $d$ ) is given by:

$$d = \frac{d}{n_{relative}}$$

$$n_{relative} = \frac{n_1 (\text{R.I. of medium of incidence})}{n_r (\text{R.I. of medium of refraction})}$$

$$\text{Apparent shift} = d \left( 1 - \frac{1}{n_{rel}} \right)$$

Refraction through a composite slab (or refraction through a number of parallel media, as seen from a medium of R.I.  $n_0$ )

Apparent depth (distance of final image from final surface)

$$= \frac{1}{2} + \frac{2}{2} + \frac{3}{3} + \dots + \dots$$

$$\text{Apparent shift} = t_1 \left(1 - \frac{1}{n_{1rel}}\right) + t_2 \left(1 - \frac{1}{n_{2rel}}\right) + \dots + 1 - \frac{1}{n_{nrel}}$$

## 6. Critical angle and total internal reflection (T.I.R.)

$$C = \sin^{-1}$$

I) Conditions of T.I.R.

- a) Light is incident on the interface from denser medium.
- b) Angle of incidence should be greater than the critical angle ( $i > c$ )

## 7. Refraction through prism

### 9.1 characteristics of a prism

$$\delta = (i + e) - (r_1 - r_2) \text{ and } r_1 + r_2 = A$$

$$\therefore \delta = i + e - A$$

### 9.2 variation of

- 1) There is one and only one angle of incidence for which the angle of deviation is minimum.
- 2) When  $\delta = \delta_{m_n}$ , the angle of minimum deviation, then  $i = e$  and  $r_1 = r_2$ , the ray passes symmetrically w.r.t. the refracting surfaces. We can show by simple calculation that  $\delta_{m_n} = 2i_{m_n} - A$ , where  $i_{m_n}$  = angle of incidence for minimum deviation and  $r = A/2$ .

$$\therefore n_{rel} = \frac{\sin \left[ \frac{A + \delta_m}{2} \right]}{\sin \left[ \frac{A}{2} \right]}, \text{ where } n_{rel} = \underline{\hspace{2cm}}$$

Also,  $\delta_{m^2n} = (n-1)A$  (for small values of A)

3) For a thin prism ( $A \ll 10^\circ$ ) and for small value of I, all values of

$$\delta = (n_{rel} - 1)A \text{ where } n_{rel} = \frac{n_{prism}}{n_{surrounding}}.$$

### 8. Dispersion of light

The angular splitting of a ray of white light into a number of components and spreading in different directions is called dispersion of light. This phenomenon is because waves of different wavelengths move with same speed in vacuum but with different speeds in a medium.

The refractive index of a medium depend slightly on wavelength also. This variation of refractive index with wavelength is given by Cauchy's formula.

Cauchy's formula  $n(\lambda) = a + \frac{b}{\lambda^2}$  Where a and b are positive constants of a medium.

Angle between the rays of the extreme colours in the refracted (dispersed) lights is called angle of dispersion.

For prism of small A and with small i :  $\theta = (n_v - n_r)A$

Deviation of beam (also called mean deviation)  $\delta = \delta_y = (n_y - 1)A$

Dispersive power ( $\omega$ ) of the medium of the material of prism is given by:

$$\omega = \frac{n_v - n_r}{n_y - 1}$$



For small angled prism ( $A \approx 10^\circ$ ) with light incident at small angle  $i =$

$$\frac{n_v - n_r}{n_y - 1} = \frac{\delta_v - n\delta_r}{\delta_y} = \frac{\theta}{\delta_y}$$

= angular dispersion

$$\left[ n_y = \frac{n_v + n_r}{2} \text{ if } n_y \text{ is not given in the problem} \right]$$

$$\omega = \frac{\delta_v - \delta_r}{\delta_y} = \frac{n_v - n_r}{n_y - 1} \left[ \text{take } n_y = \frac{n_v + n_r}{2} \text{ if value of } n_y \text{ is not given in the problem} \right]$$

$n_v, n_r$  &  $n_y$  are R.I of material for violet, red and yellow colors respectively.

### 9. Combination of 2 prisms

Two or more prisms can be combined in various ways to get different combination of angular dispersion and deviation.

a) Direct vision combination (dispersion without deviation)

The condition for direct vision combination is

$$\frac{n_v - n_r}{2} - 1 A = \frac{n_v + n_r}{2} - 1 A [n_y - 1] A = [n_y - 1] A$$

b) Achromatic combination (deviation without dispersion)

Condition for achromatic combination is

$$(n_v - n_r) A = (n_v - n_r) A$$

### 10. Refraction at spherical surfaces

For paraxial rays incident on a spherical surface separating two media:

Where light moves from the medium of refractive index  $n_1$  to the medium of refractive index  $n_2$ .

Transfers magnification ( $m$ ) (of dimension perpendicular to principal axis) due to refraction at spherical surface is given by

$$m = \frac{v - R}{u - R} = \frac{v/n_2}{u/n_1}$$

### 11. Refraction at spherical thin lens

A thin lens is called convex if it is thicker at the middle and it is called concave if it is thicker at the ends.

For a spherical, thin lens having the same medium on both sides:

$$\frac{1}{v} - \frac{1}{u} = (n_{rel} - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \text{ where } n_{rel} = \frac{n_2}{n_1}$$

$$\frac{1}{f} = (n_{rel} - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$$

Lens maker's formula  $\frac{1}{f} = (n_{rel} - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$

Combination of lenses  $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} \dots$

### Modern physics

- Work function is minimum for cesium (1.9 eV)
- Work function  $W = h\nu_0 = \frac{hc}{\lambda_0}$
- Photoelectric current is directly proportional to intensity of incident radiation. ( $\nu$  constant)
- Photoelectron ejected from metal have kinetic energies ranging from 0 to  $K_{Emax} = eV_s$

Here  $K_{Emax} = eV_s$

$V_s$  – stopping potential

- Stopping potential is independent of intensity of light used ( $v - \text{constant}$ )
- Intensity in terms of electric field is

$$I = \frac{1}{2\epsilon_0} E^2 \cdot c$$

- Momentum of one photon is  $\frac{h}{\lambda}$
- Einstein equation of photoelectric effect is  $h\nu = w_0 + k_{\max}$       $\frac{hc}{\lambda} = \frac{hc}{\lambda_0} + eVs$
- Energy  $\Delta E = \frac{12400}{\lambda(A^{\circ})} eV$
- Force due to radiation (photon) (no transmission)

When light is incident perpendicularly

(a)  $a=1$   $r=0$

$$F = \frac{IA}{c}, \text{ pressure} = \frac{I}{c}$$

(b)  $r=1$ ,  $a=0$

$$F = \frac{2IA}{c}, P = \frac{2I}{c}$$

(c) when  $0 < r < 1$  and  $a+r=1$

$$F = \frac{IA}{c}(1+r), P = \frac{1}{c}(1+r)$$

When light is incident at an angle with vertical

$a=1$ ,  $r=0$

(a)  $F = \frac{IA \cos \theta}{c}$ ,  $P = \frac{F \cos \theta}{A} = \frac{I}{c} \cos 2\theta$

$r=1$ ,  $a=0$

(b)  $F = \frac{2IA \cos^2 \theta}{c}$ ,  $P = \frac{2I \cos^2 \theta}{c}$

$$0 < r < 1, a+r=1$$

$$(c) P = \frac{I \cos^2 \theta}{c} (1+r)$$

- De Broglie wavelength  $\lambda = \frac{h}{mv} = \frac{h}{P} = \frac{h}{\sqrt{2km}}$

- Radius and speed of electron in hydrogen like atoms

$$r_n = \frac{n^2}{Z} a_0, \quad a_0 = 0.529 \text{ \AA}$$

$$v_n = \frac{Z}{n} v_0, \quad v_0 = 2.19 \times 10^6 \text{ m/s}$$

- Energy in nth orbit  $E_n = E_1 \cdot \frac{Z^2}{n^2}$      $E_1 = 13.6 \text{ eV}$

- Wavelength corresponding to spectral lines

$$\frac{1}{\lambda} = R \left( \frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

For Lyman series     $n_1 = 1$      $n_2 = 2, 3, 4, \dots$

Balmer     $n_1 = 2$      $n_2 = 3, 4, 5, \dots$

Paschen     $n_1 = 3$      $n_2 = 4, 5, 6, \dots$

- The Lyman series is an ultraviolet and Paschen, Brackett and Pfund series are in the infrared region.
- Total number of possible transitions, is  $n(n-1)/2$ . (from nth state)
- If effect of nucleus motion is considered,

$$r_n = (0.529 \text{ \AA}) \frac{n^2}{Z} \cdot \frac{m}{\mu}$$

Here  $\mu$  - reduced mass

$$\mu = \frac{Mm}{(M+m)}, M - \text{mass of nucleus}$$

- Minimum wavelength of X rays

$$\lambda_{\min} = \frac{hc}{eV_0} = \frac{12400}{V_0(\text{volt})} \text{ \AA}$$

- Moseley's law

$$\sqrt{\nu} = a(z - b)$$

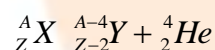
a and b are positive constants for one type of X rays (independent of z)

- Average radius of nucleus may be written as

$$R = R_0 A^{1/3}, R_0 = 1.1 \times 10^{-15} \text{ M}$$

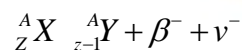
A – mass number

- Binding energy of nucleus of mass M, is given by  $B = (ZM_p + NM_n - M)C^2$
- Alpha – decay process



$$Q\text{-value} = Q = [m({}^A_Z X) - m({}^{A-4}_{Z-2} Y) - m({}^4_2 \text{He})] C^2$$

- Beta plus-decay



$$Q\text{-value} = Q = [m({}^A_Z X) - m({}^A_{Z-1} Y) - m({}^0_{-1} e)] c^2$$

- Electron capture: when atomic electron is captured, x-rays are emitted.

$$A = A_0 e^{-\lambda t}$$

- In radioactive decay, number of nuclei at instant  $t$  is given by  

$$N = N_0 e^{-\lambda t}, \lambda - \text{decay constant.}$$
- Activity of sample:  $A = A_0 e^{-\lambda t}$
- Activity per unit mass is called specific activity.
- Half life:  $T_{1/2} = \frac{0.693}{\lambda}$
- Average life:  $T_{av} = \frac{T_{1/2}}{0.693}$
- A radioactive nucleus can decay by 2 different processes having half lives  $t_1$  and  $t_2$  respectively. Effective half life of nucleus is given by  $1/t = 1/t_1 + 1/t_2$

### Wave optics

- Interference of waves of intensity  $I_1$  and  $I_2$ :
- Resultant intensity,  $I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos(\Delta\phi)$ , where  $\Delta\phi =$  phase difference.
- For constructive interference:  $I_{\max} = (\sqrt{I_1} + \sqrt{I_2})^2$
- For destructive interference:  $I_{\min} = (\sqrt{I_1} - \sqrt{I_2})^2$
- If sources are incoherent  $I = I_1 + I_2$ , at each point.
- YDSE:

Path difference,  $\Delta p = S_2 P - S_1 P = d \sin \theta$

If  $d \ll D = \frac{dy}{D}$

If  $y \ll D$

❖ For maxima,  $\Delta p = n\lambda$ ,  $y = n\beta$   $n = 0, \pm 1, \pm 2, \dots$

$$(2n+1)\frac{\lambda}{2} \quad n=1,2,3,\dots$$

❖ For minima,  $\Delta p = \Delta p = (2n+1)\frac{\lambda}{2}$   $n=-1,-2,-3,\dots$

$$y = (2n-1)\frac{\beta}{2} \quad n=1,2,3,\dots$$

$$(2n+1)\frac{\beta}{2} \quad n=-1,-2,-3,\dots$$

Where, fringe width  $\beta = \frac{\lambda D}{d}$

Here  $\lambda$  is wavelength in medium.

- Highest order maxima:  $n_{\max} = \frac{d}{\lambda}$
- Total number of maxima:  $2n_{\max} + 1$
- Highest order minima:  $n_{\max} = \frac{d}{\lambda} + \frac{1}{2}$

Total number of minima:  $2n_{\max}$

Intensity on screen:  $I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos(\Delta\phi)$ , where  $\Delta\phi = \frac{2\pi}{\lambda} \Delta p$

If  $I_1 = I_2, I = 4I_1 \cos^2 \frac{\Delta\phi}{2}$

- YDSE with two wave lengths  $\lambda_1$  &  $\lambda_2$

The nearest point to central maxima where the bright fringes coincide:

$$y = n_1\beta_1 = n_2\beta_2 = \text{LCM of } \beta_1 \text{ and } \beta_2$$

The nearest point to central maxima where the two dark fringes coincide,

$$y = \left( n \frac{1}{2} \right) \beta = n \frac{1}{2} \beta$$

➤ Optical path difference

$$\Delta p_{opt} = \mu \Delta p$$

$$\Delta \phi = \frac{2\pi}{\lambda} \Delta p = \frac{2\pi}{\lambda_{vacuum}} \Delta p_{opt}$$

$$\Delta = (\mu - 1)t \cdot \frac{D}{d} = (\mu - 1)t \frac{\beta}{\lambda}$$

➤ YDSE with oblique incidence

In YDSE, ray is incident on the slit at an inclination of  $\theta_0$  to the axis of symmetry of the experimental set-up.

We obtain central maxima at a point where,  $\Delta p = 0$  or  $\theta = \theta_0$

This corresponds to the point O in the diagram.

Hence we have path difference.

$$\Delta p = d(\sin \theta_0 + \sin \theta) - \text{for points above O}$$

$$= d(\sin \theta_0 - \sin \theta) - \text{for points between O \& O'}$$

$$= d(\sin \theta_0 - \sin \theta) - \text{for points below O}$$

➤ **Thin-film interference**

For interference in reflected light:  $2\mu d = \frac{n\lambda}{\left(n + \frac{1}{2}\right)}$

For destructive interference

For constructive interference

For interference in transmitted light:  $2\mu d = \frac{n\lambda}{\left(n + \frac{1}{2}\right)}$

For destructive interference



For constructive interference

## Gravitation

$$\diamond F = \frac{m_1 m_2}{r^2} \text{ or } F = G \frac{m_1 m_2}{r^2}$$

Where  $G = 6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$  is the universal gravitational constant

➤ Newton's law of gravitation in vector form:

$$F_{12} = \frac{Gm_1 m_2}{r^2} \hat{r}_{12} \text{ \& } F_{21} = \frac{Gm_1 m_2}{r^2} \hat{r}_{21}$$

➤ Gravitational field:

$$E = F/M = GM/r^2$$

➤ Gravitational potential:

$$\text{Gravitational potential, } V = GM/r \quad E = dV/Dr$$

$$1. \text{ Ring } V = \frac{-GM}{x \text{ or } (a^2 + r^2)^{1/2}} \text{ \& } E = \frac{-GMr}{(a^2 + r^2)^{3/2}} \text{ or } E = \frac{GM \cos \theta}{x^2}$$

Gravitational field is maximum at a distance

2. Thin circular disc.

$$V = \frac{-2GM}{a^2} [a^2 + r^2]^{1/2} - r \text{ and } E = \frac{2GM}{a^2} \left[ 1 - \frac{r}{[r^2 + a^2]^{1/2}} \right] = \frac{2GM}{a^2} [1 - \cos \theta]$$

3. A) point P inside the sphere  $r \leq a$ , then

$$V = -\frac{GM}{2a^3} (3a^2 - r^2) \text{ \& } E = \frac{GMr}{a^3}, \text{ and at the centre } V = \frac{3GM}{2a} \text{ and } E=0$$

B) point P outside the sphere  $r \geq a$ , then  $V = -\frac{GM}{r} \text{ \& } E = \frac{GM}{r^2}$

#### 4. Uniform thin spherical shell

a) Point P inside the shell  $r \leq a$ , then  $V = -\frac{GM}{a}$  &  $E = 0$

b) Point P outside shell  $r \geq a$  then,  $V = -\frac{GM}{r}$  &  $E = \frac{GM}{r^2}$

Variation of acceleration due to gravity:

1. Effect of altitude  $g_h = \frac{GM_e}{(R_e + h)^2} = g \left(1 + \frac{h^2}{R_e^2}\right)^{-1} \approx g \left(1 - \frac{2h}{R_e}\right)$  when  $h \ll R_e$ .

2. Effect of depth:  $g_0 = g \left(1 - \frac{d}{R_e}\right)$

3. Effect of the surface of earth

The Equatorial radius is about 21 kilometre longer than its polar radius.

We know,  $g = \frac{GM_e}{R_e^2}$  hence  $g_{pole} \gg g_{equation}$

Satellite velocity (or orbital velocity)

$$v_0 = \frac{GM_e^{1/2}}{(R_e + h)} = \frac{gR_e^2}{(R_e + h)^{1/2}}$$

When  $h \ll R_e$  then  $v_0 = \sqrt{gR_e}$

$$\therefore V_0 = \sqrt{9.8 \times 6.4 \times 10^6} = 7.92 \times 10^3 \text{ ms}^{-1} = 7.92 \text{ km s}^{-1}$$

$$\text{Time period of satellite: } T = \frac{2\pi(R_e + h)}{\frac{gR_e^2}{(R_e + h)^{1/2}}} = \frac{2\pi(R_e + h)^{3/2}}{R_e g}$$

$$\text{Energy of a satellite } U = \frac{-GM_e m}{r} \quad \text{K.E} = \frac{GM_e m}{2r}; \text{ then total energy } E = \frac{GM_e m}{2R_e}$$

Kepler's laws

Law of area:

The line joining the sun and a planet sweeps out equal areas in equal intervals of time.

Area velocity = area swept/ time =  $\frac{\frac{1}{2}r(rd\theta)}{dt} = \frac{1}{2}r^2 \frac{d\theta}{dt} = \text{constant}$ . Hence  $\frac{1}{2}r^2\omega =$   
constant

Law of periods:  $\frac{T^2}{R^3} = \text{constant}$

### Fluid mechanics & properties of matter

Fluids, surface tension, viscosity and elasticity:

1. Hydraulic press  $p = \frac{f}{a} = \frac{F}{A}$  or  $F = \frac{A}{a} f$

Hydrostatic paradox  $P_A = P_B = P_C$

- I) Liquid placed in elevator: when elevator accelerates upward with acceleration  $a_0$  then pressure in the fluid, at depth  $h$  may be given by  $p = \rho h[g + a_0]$

And force of buoyancy,  $B = m(g + a_0)$

- II) Free surface of liquid in horizontal acceleration:  $\tan \theta = \frac{a_0}{g}$

$p_1 p_2 = \rho \ell a_0$  where  $p_1$  and  $p_2$  are pressure at points 1 & 2 . then  $h_1 h_2 = \frac{\ell a_0}{g}$

- III) Free surface of liquid in case of rotating cylinder.  $h = \frac{v^2}{2g} = \frac{\omega^2 r^2}{2f}$

Equation of continuity  $a_1v_1 = a_2v_2$

In general,  $av = \text{constant}$ .

Bernoulli's theorem

i.e.  $\frac{P}{\rho} + \frac{1}{2}v^2 + gh = \text{constant}$

vi) Torricelli's theorem (speed of efflux)  $v = \sqrt{\frac{2gh}{1 - \frac{A_2^2}{A_1^2}}}$ ,  $A_2 = \text{area of hole}$   $A_1 = \text{area}$

of vessel.

Elasticity and viscosity:

stress = restoring force/area of the body =  $F/A$

strain = change in configuration/original configuration

I) Longitudinal strain =  $\frac{\Delta L}{L}$

II) Volume strain  $\frac{\Delta V}{V}$

III) Shear strain  $\tan \phi$  or  $\phi = \frac{x}{\ell}$

1. Young's modulus of elasticity  $Y = \frac{F/A}{\Delta L/L} = \frac{FL}{A\Delta L}$

➤ Potential energy per unit volume  $\frac{1}{2}(\text{stress strain}) = \frac{1}{2}(Y \text{ strain})^2$

➤ Inter-atomic force-constant  $k = Yr_0$

➤ Newton's law of viscosity  $F = A \frac{dv}{dx}$  or  $F = \eta A \frac{dv}{dx}$

➤ Stoke's law  $F = 6\pi\eta rv$

➤ Terminal velocity  $= \frac{2}{9} \frac{r^2(\rho - \sigma)}{\eta}$

➤ Surface tension

Surface tension (T) = Total force on either of the imaginary line (F)/ Length

of the line  $\ell$ ;  $T = S = \frac{\Delta W}{A}$

❖ Thus, surface tension is numerically equal to surface energy or work done per unit increase surface area.

➤ Inside a bubble:  $(p - p_a) = \frac{4T}{r} = p_{excess}$ ;

➤ Inside the drop:  $(p - p_a) = \frac{2T}{r} = p_{excess}$ ;

➤ Inside air bubble in a liquid:  $(p - p_a) = \frac{2T}{r} = p_{excess}$

➤ Capillary rise  $h = \frac{2T \cos \theta}{r \rho g}$

## Sound waves

I) Longitudinal displacement of sound wave

$$\xi = A \sin(\omega t - kx)$$

II) Pressure excess during travelling sound wave

$$P_{ex} = -B \frac{\partial \xi}{\partial x} \text{ (it is true for travelling)}$$

$$= (BAk) \cos(\omega t - kx)$$

Amplitude of pressure excess =  $BAk$

III) Speed of sound  $C = \sqrt{\frac{E}{\rho}}$

Where, E = elastic modulus for the medium

$\rho$  = density of medium

For solid  $C = \sqrt{\frac{Y}{\rho}}$

Where Y = young's modulus for the solid

For liquid  $C = \sqrt{\frac{B}{\rho}}$

Where, B = bulk modulus for the liquid

For gases  $C = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M_0}}$

Where  $M_0$  is molecular wt. of the gas in (kg/mole)

Intensity of sound wave:

$$\langle I \rangle = 2\pi^2 f^2 A^2 \rho v = \frac{P_m^2}{2\rho v} \quad \langle I \rangle = P_m^2$$

IV) Loudness of sound:  $L = 10 \log_{10} \frac{I}{I_0} \text{ dB}$

Where  $I_0 = 10^{-12} \text{ W/m}^2$

(this the minimum intensity human ears can listen)

Intensity at a distance r from a point source  $I = \frac{P}{4\pi r^2}$

Interference of sound wave

If  $P_1 = p_{m1} \sin(\omega t - kx_2 + \theta_1)$   
 $P_2 = p_{m2} \sin(\omega t - kx_2 + \theta_2)$

Resultant excess pressure at point O is

$$p = p_1 + p_2$$

$$p = p_0 \sin(\omega t - kx + \theta)$$

$$p_0 = \sqrt{p_{m1}^2 + p_{m2}^2 + 2p_{m1}p_{m2} \cos \phi}$$

$$\text{Where } \phi = [k(x_1 - x_2) + (\theta_1 - \theta_2)]$$

$$\text{And } I = I_1 + I_2 + 2\sqrt{I_1 I_2}$$

I) For constructive interference

$$\phi = 2n\pi \text{ and } p_0 = p_{m1} + p_{m2} \text{ (constructive interference)}$$

II) For destructive interference

$$\phi = (2n+1)\pi \text{ and } p_0 = |p_{m1} - p_{m2}| \text{ (destructive interference)}$$

If  $\phi$  is due to path difference only then  $\phi = \frac{2\pi}{\lambda} \Delta x$

Condition for constructive interference:  $\Delta x = n\lambda$

Condition for destructive interference:  $\Delta x = (2n+1)\frac{\lambda}{2}$

(a) If  $p_{m1} = p_{m2}$  and  $\theta = \pi, 3\pi, \dots$

Resultant  $p=0$  i.e. no sound

(b) If  $p_{m1} = p_{m2}$  and  $\phi = 0, 2\pi, 4\pi \dots$

$$p_0 = 2p_m \text{ \& } I_0 = 4I_1$$

$$p_0 = 2p_{m1}$$

Close organ pipe:

$$f = \frac{v}{4\ell}, \frac{3v}{4\ell}, \frac{5v}{4\ell}, \dots, \frac{(2n+1)v}{4\ell} \quad n = \text{overtone}$$

Open organ pipe:

$$f = \frac{v}{2l}, \frac{2v}{2l}, \frac{3v}{2l}, \dots, \frac{nV}{2l}$$

Beats: beats frequency =  $|f_1 - f_2|$

Doppler's effect

The observed frequency,  $f = f \frac{V - V_0}{V - V_s}$  a

And apparent wavelength  $\lambda = \lambda \frac{V - V_s}{V}$