Revision Notes

Class - 11 Maths

Chapter 16 - Probability

- **Probability** is a numerical measure of the uncertainty of diverse phenomena. It can range from 0 to 1 a positive value.
- The phrases 'probably', 'doubt', 'most probably', 'chances' and so on, all have an element of ambiguity in them.
- Probability$=\dfrac{\text{no. of favourable outcome}}{\text{total no. of outcomes}}$
- **Approaches to Probability:**

i. Statistical approach: Observation & data collection

ii. Classical approach: Only Equal probable events

iii. Axiomatic method: For real-life situations. It has a strong connection to set theory.

- **Random Experiments:**

(i) There are multiple possible outcomes.

(ii) It is impossible to know the outcome ahead of time.

- **Outcomes:** An outcome is a probable result of a random experiment.

- **Sample space** refers to the set of all possible results of a random experiment. The letter S stands for it. For example, in a coin toss, the sample space is Head, Tail.

  Each element of the sample space is referred to as a **sample point**. For example, in a coin flip, the head is a sample point.

- **Event:**

An event is a collection of favourable outcomes.

An event is defined as a subset E of a sample space S. For example, suppose you get an unusual result when you roll a dice.

- **Occurrence of an event:**

The occurrence of an event E in a sample space S is said to have occurred if the experiment's outcome ω is such that $\omega \in E$. We say that the event E did not happen if the outcome ω is such that $\omega \in E$.

- **Types of Event**

i. Impossible and Sure Events

ii. Simple Event

iii. Compound Event

- **Impossible and Sure Events:**

  Events that are both impossible and certain are described by the empty set ϕ and the sample space S. The impossible event is denoted by ϕ, and the entire sample space is referred to as the Sure Event.

  For example, while rolling a dice, an impossible event is when the number is greater than 6 and a sure event is when the number is less than or equal to 6.

- **Simple (or elementary) event:** A simple event has only one sample point of a sample space.

  There are exactly n simple occurrences in a sample space with n different items. For example, if you roll a dice, a simple event could be receiving a four.

- **Compound Event:** A compound event is one in which there are multiple sample points.

  For example, in the case of rolling a die, a simple event could be the event of receiving a four.

- **Algebra of Events:**

i. Complementary Event

ii. Event 'A or B'

iii. Event 'A and B'

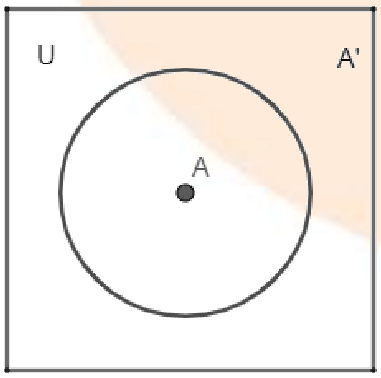
iv. Event 'A but not B

- **Complementary Event**

Complementary event to A='not A'

Example: If event A= Event of getting odd number in throw of a die, that is $\{1, 3, 5\}$ Then, Complementary event to A= Event of getting even number in throw of a die, that is $\{2, 4, 6\}$

$A' = \{\omega : \omega \in S \text{ and } \omega \notin A\} = S - A$ (Where S is the Sample Space)



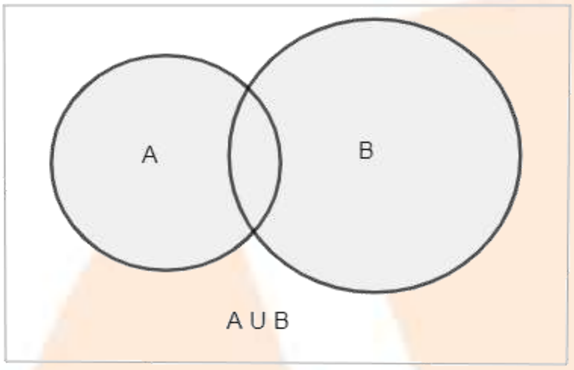
- **Event (A or B):**

$A \cup B$ is known as the union of two sets A and B, it contains all those elements which are present in either of the two sets.

If the sets A and B correspond to two events in a sample space, then '$A \cup B$' is the event 'either A or B or both'. This event '$A \cup B$' is also called 'A or B'

Event

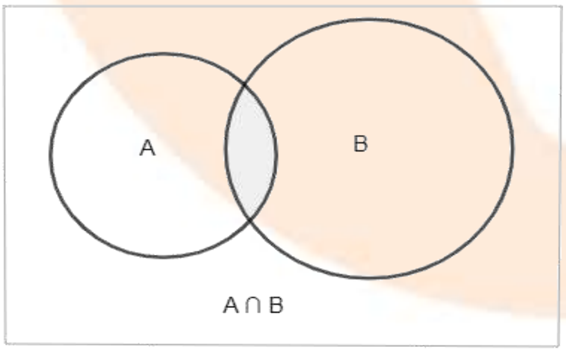
$A \text{ or } B = A \cup B = \{ \omega : \omega \in A \text{ or } \omega \in B \}$



- **Event 'A and B' :**

$A \cap B$ is known as the intersection of two sets A and B, it contains all those elements which are common in both the two sets. i.e., which belong to both 'A and B'. If A and B are two events, then the set $A\,A \cap B$ denotes the event 'A and B'.

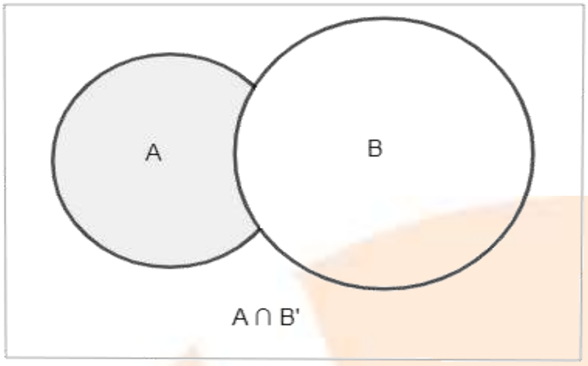
Thus, $A \cap B = \{\omega : \omega \in A \text{ and } \omega \in B\}$



- **Event 'A but not B'**

$A–B$ is the set of all those elements which are in A but not in B . Therefore, the set $A–B$ may denote the event ' A but not B '.

$A - B = A \cap B'$



- **Mutually exclusive events**

Events A and B are said to be mutually exclusive if the occurrence of one of them precludes the occurrence of the other, i.e., if they can't happen at the same time.

A die is thrown, for example. All even outcomes is event A, and all odd outcomes is event B. Then A and B are mutually exclusive events, and they cannot happen at the same time.

A sample space's simple events are always mutually exclusive.

- **Exhaustive events:**

Sample space contains lot of events together.

Example: A die is thrown.

Event $A=$ All even outcome and event $B=$ All odd outcome. Even A & B together forms exhaustive events as it forms Sample Space.

- **Axiomatic Approach to Probability:**

Another way of explaining probability by using axioms are rules is called the Axiomatic approach.

Let S be sample space of a random experiment. The probability P is a real valued function whose domain is the power set of S and range is the interval $[0,1]$ satisfying the following axioms

I. For any event E, $P[E] \geq 0$

II. $P[S] = 1$

III. If E and F are mutually exclusive events, then $P(E \cup F) = P(E) + P(F)$

It follows from (III) that $P(\phi)=0$. Let $F=\phi$ and $E=\phi$ be two disjoint events,

$\therefore P(E \cup \phi) = P(E) + P(\phi)$ or $P(E) = P(E)+P(\phi)$ i.e $P(\phi)=0$

Let S be a sample space containing outcomes $\omega_1, \omega_2, ...\omega_n$ i.e., $S = \{\omega_1, \omega_2, ..., \omega_n\}$ then

I. $0 \leq P(\omega_i) \leq 1$ for each $\omega_i \in S$

II. $P(\omega_1) + P(\omega_2) + ... + P(\omega_n) = 1$

III. For any event $E, P(E) = \sum P(\omega_i), \omega_i \in A$

IV. $P(\phi) = 0$

- **Probabilities of equally likely outcomes**:

Let $P(\omega_i) = p$, for all $\omega_i \in S$ where $0 \leq p \leq 1$, then $p = \frac{1}{n}$ where n= number of elements.

Let S be a sample space and E be an event, such that $n(S)=n$ and $n(E) = m$. If each outcome is equally likely, then it follows that $P(E)=\frac{m}{n}$

$$= \frac{\text{Number of outcomes favourable to E}}{\text{Total possible outcomes}}$$

- **Probability of the event ' A or B' :**

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

- **Probability of the event 'A and B' :**

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

- **Probability of the event 'Not A '**

$P(A') = P(\text{not } A) = 1 - P(A)$