Revision Notes

Class – 11 Mathematics

Chapter 5 - Complex Number and Quadratic Equations

1. Definition

When a given number is in the form of $a+ib$, where $a,b \in R$ and $i=\sqrt{-1}$ it is called a complex number and such number is denoted by 'z'.

$z=a+ib$

Where,

a = real part of complex number and,

b = imaginary part of complex number.

1.1 Conjugate of a Complex Number

Consider a complex number $z=a+ib$,

Then its conjugate is written as '$\bar{z}$'.

Whose value is defined as $\bar{z}=a-ib$.

2. ALGEBRA OF COMPLEX NUMBERS

Let $z_1=a+ib$ and $z_2=c+id$ be two complex numbers where $a,b,c,d \in R$ and $i=\sqrt{-1}$.

1. Addition:

$$\begin{aligned} z_1+z_2 &= (a+bi)+(c+di) \\ &= (a+c)+(b+d)i \end{aligned}$$

2. Subtraction:

$$
\begin{aligned}
z_1 - z_2 &= (a+bi)-(c+di) \\
&= (a-c)+(b-d)i
\end{aligned}
$$

3. Multiplication:

$$
\begin{aligned}
z_1 \cdot z_2 &= (a+bi)(c+di) \\
&= a(c+di)+bi(c+di) \\
&= ac+adi+bci+bdi^2 \\
&= ac-bd+(ad+bc)i \\
&\quad (\because i^2 = -1)
\end{aligned}
$$

Note:

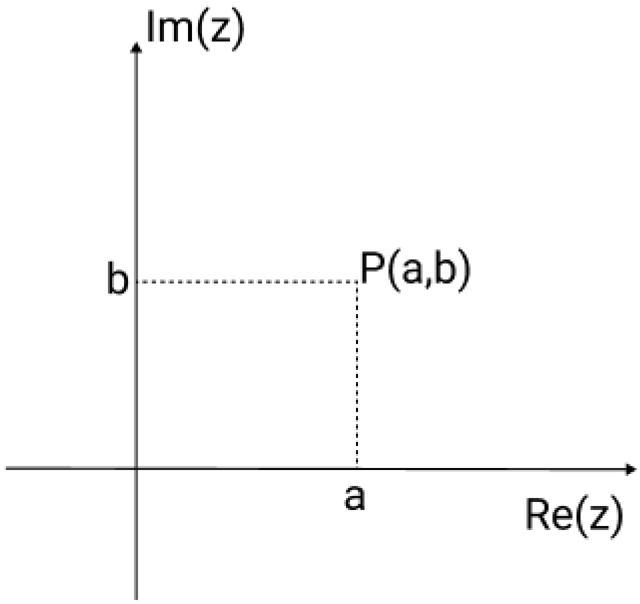
1. $a+ib = c+id$
$\Leftrightarrow a = c \backslash \quad b = d$

2. $i^{4k+r} = \begin{cases} 1; & r=0 \\ i; & r=1 \\ -1; & r=2 \\ -i; & r=3 \end{cases}$

3. $\sqrt{b}\sqrt{a} = \sqrt{ba}$ is only possible if atleast one of either a or b is non-negative.

3. ARGAND PLANE

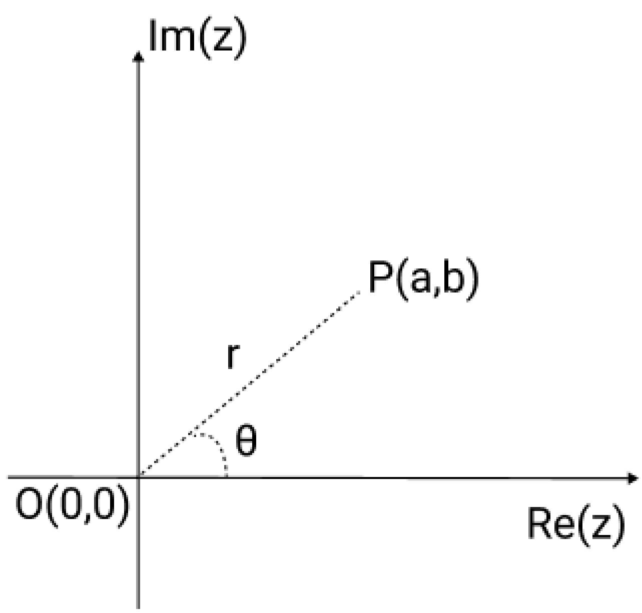
Any complex number $z = a+ib$ can be represented by a unique point $P(a,b)$ in th argand plane.



$P(a,b)$ represents the complex number $z = a + ib$.

3.1 Modulus and Argument of Complex Number

Consider a complex number $z = a + ib$.



(i) Distance of z from origin is reffered as modulus of complex number z. It is represented by $r = |z| = \sqrt{a^2 + b^2}$

(ii) Here, θ i.e.,the angle made by ray OP with positive direction of real axis is called argument of $\mathbf{z}$.

Note:

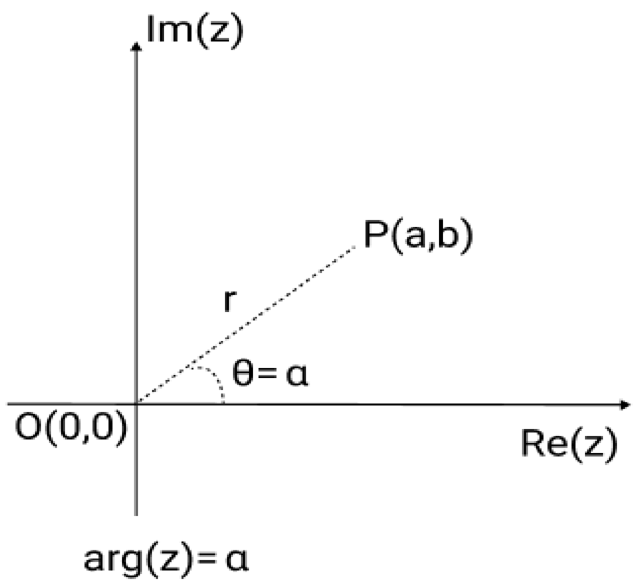
$z_1 > z_2$ or $z_1 < z_2$ has no meaning but $|z_1| > |z_2|$ or $|z_1| < |z_2|$ holds meaning.

3.2 Principal Argument

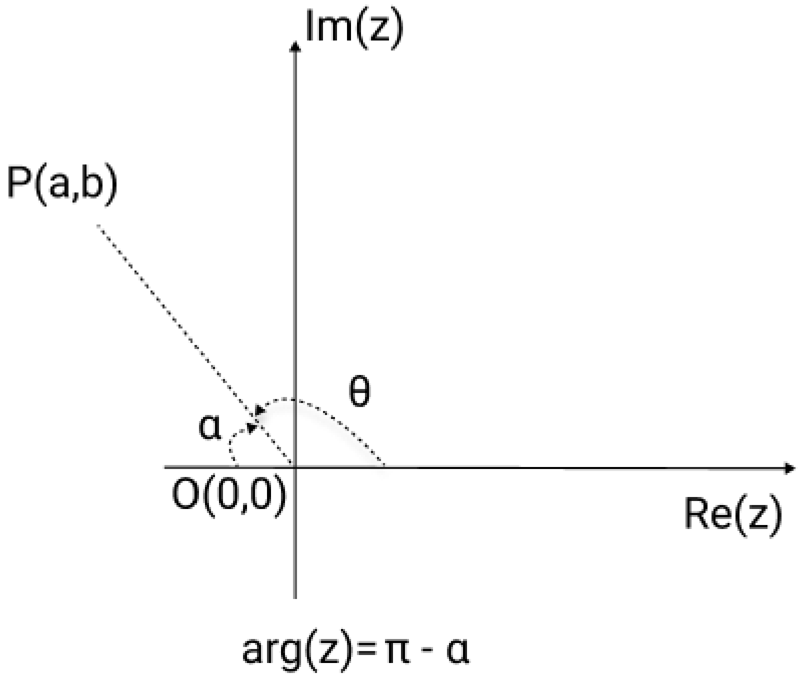
The argument $'\theta'$ of complex number $z = a + ib$ is called the principal argument of z if $-\pi < \theta \leqslant \pi$.

Consider $\tan\alpha = \left|\frac{b}{a}\right|$, and θ be the $\arg(z)$.

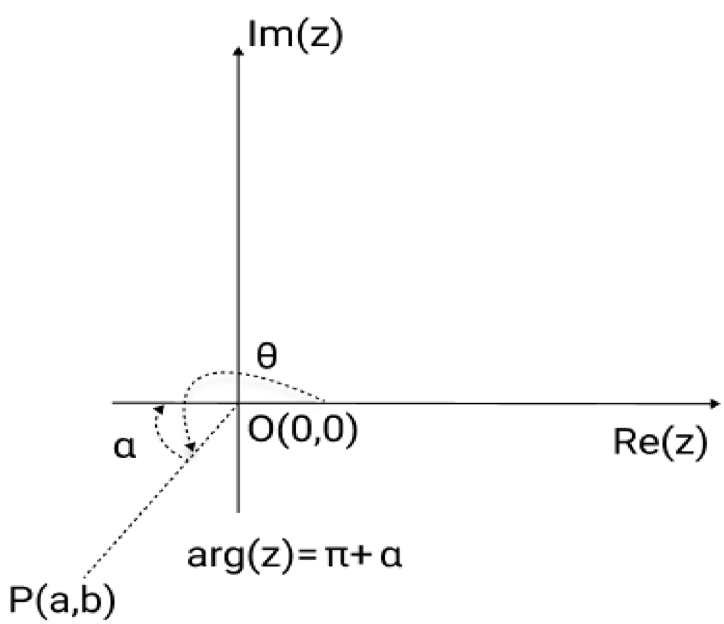
i.



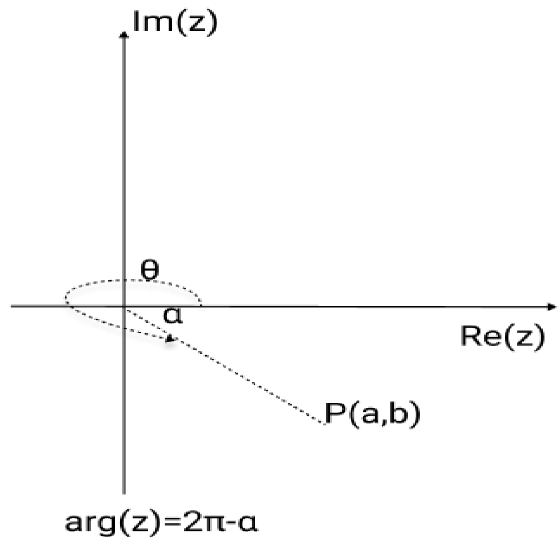
ii.



iii.

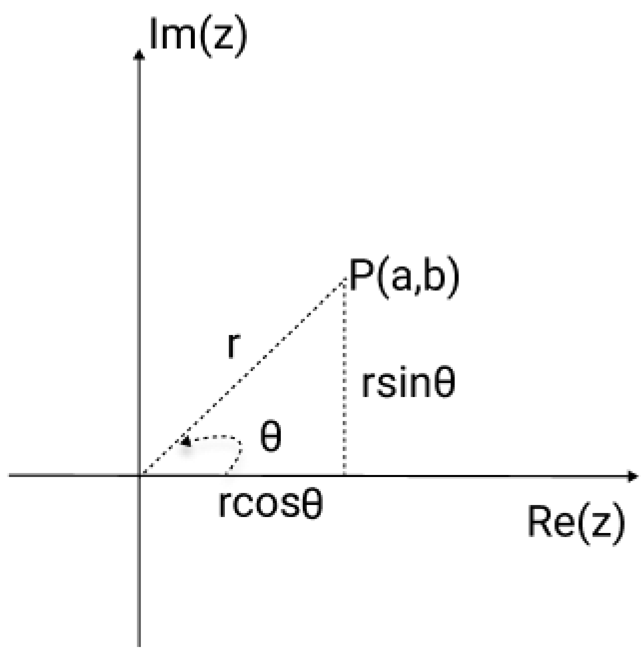


iv.



In (iii) and (iv) the principal argument is given by $-\pi+\alpha$ and $-\alpha$ respectively.

4. POLAR FORM



$a = r\cos\theta \qquad b = r\sin\theta$

where $r = |z|$ and $\theta = \arg(z)$

$\therefore \quad z = a + ib$

$= r(\cos\theta + i\sin\theta)$

Note:

A complex number z can also be represented as $z = re^{i\theta}$, it is known as Euler's form.

Where,

$r = |Z| \quad \theta = \arg(Z)$

5. SOME IMPORTANT PROPERTIES

1. $\overline{(\overline{z})} = z$

2. $z + \overline{z} = 2\,\text{Re}(z)$

3. $z - \overline{z} = 2i\,\text{Im}(z)$

4. $\overline{z_1 + z_2} = \overline{z}_1 + \overline{z}_2$

5. $\overline{z_1 z_2} = \overline{z}_1 . \overline{z}_2$

6. $|z| = 0 \Rightarrow z = 0$

7. $z.\overline{z} = |z|^2$

8. $|z_1 z_2| = |z_1||z_2|; \left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$

9. $|\overline{z}| = |z| = |-z|$

10. $|z_1 \pm z_2|^2 = |z_1|^2 + |z_2|^2 \pm 2\,\text{Re}(z_1\overline{z}_2)$

11. $|z_1 + z_2| \leqslant |z_1| + |z_2|$ (Triangle Inequality)

12. $|z_1 - z_2| \geqslant ||z_1| - |z_2||$

13. $|az_1 - bz_2|^2 + |bz_1 + az_2|^2 = (a^2 + b^2)(|z_1|^2 + |z_2|^2)$

14. $\text{amp}(z_1 \cdot z_2) = \text{amp}\, z_1 + \text{amp}\, z_2 + 2k\pi;\ k \in I$

15. $\text{amp}\left(\frac{y_0}{y_1}\right) = \text{amp}\, z_1 - \text{amp}\, z_2 + 2k\pi;\ k \in I$

16. $\text{amp}(z^n) = n\,\text{amp}(z) + 2k\pi;\ k \in I$

6. DE-MOIVRE'S THEOREM

Statement: $\cos n\theta + i\sin n\theta$ is the value or one of the values of $(\cos\theta + i\sin\theta)^n$ according as if 'n' is integer or a rational number. The theorem is very useful in determining the roots of any complex quantity.

7. CUBE ROOT OF UNITY

Roots of the equation $x^3 = 1$ are called cube roots of unity.$

Roots of the equation $x^3 = 1$ are called cube roots of unity.

$$x^3 - 1 = 0$$
$$(x-1)(x^2 + x + 1) = 0$$
$$x = 1 \quad \text{or} \quad x^2 + x + 1 = 0$$

i.e $x = \underbrace{\frac{-1+\sqrt{3}i}{2}}_{w}$ **or** $x = \underbrace{\frac{-1-\sqrt{3}i}{2}}_{w^2}$

(i) The cube roots of unity are $1, \frac{-1+i\sqrt{3}}{2}, \frac{-1-i\sqrt{3}}{2}$.

(ii) $\omega^3 = 1$

(iii) If w is one of the imaginary cube roots of unity then $1+\omega+\omega^2 = 0$.

(iv) In general $1+\omega^{r}+\omega^{2r} = 0$; where $r \in I$ but is not the multiple of 3 .

(v) In polar form the cube roots of unity are:

$$\cos 0 + i\sin 0; \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}, \cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}$$

(vi) The three cube roots of unity when plotted on the argand plane constitute the vertices of an equilateral triangle.

(vii) The following factorisation should be remembered:

$$a^3 - b^3 = (a-b)(a-\omega b)\left(a-\omega^2 b\right)$$
$$x^2 + x + 1 = (x-\omega)\left(x-\omega^2\right)$$
$$a^3 + b^3 = (a+b)(a+\omega b)\left(a+\omega^2 b\right)$$
$$a^3 + b^3 + c^3 - 3abc = (a+b+c)\left(a+\omega b+\omega^2 c\right)\left(a+\omega^2 b+\omega c\right)$$

8. ‘n’ nth ROOTS OF UNITY

Solution of equation $x^n = 1$ is given by,

$$x = \cos\frac{2k\pi}{n} + i\sin\frac{2k\pi}{n} \quad ; k = 0, 1, 2, \ldots, n-1$$
$$= e^{i\left(\frac{2k\pi}{n}\right)} \quad ; k = 0, 1, \ldots, n-1$$

Note:

1. We may take any n consecutive integral values of k to get ' n ' n^{th} roots of unity.

2. Sum of ' n' n^{th} roots of unity is zero, $n \in N$

3. The points represented by 'n', nth roots of unity are located at the vertices of regular polygon of n sides inscribed in a unit circle, centred at origin and one vertex being one positive real axis.

Properties:

If $1, \alpha_1, \alpha_2, \alpha_3 \ldots \alpha_{n-1}$ are the n, n^{th} root of unity then:

(i) They are in G.P. with common ratio $e^{i(2\pi/n)}$

(ii) $1^p + \alpha_0^0 + \alpha\_1\wedge\wedge^\circ + \ldots + \alpha\_m - 0\wedge\wedge^\circ = \begin{cases} 0, & \text{if } p \neq k^n \\ n, & \text{if } p = kn \end{cases}$ where $k \in Z$

(iii) $(1-\alpha_1)(1-\alpha_2)\ldots\ldots(1-\alpha_{n-1}) = n$

(iv) $(1+\alpha_1)(1+\alpha_2)\ldots\ldots(1+\alpha_{n-1}) = \begin{cases} 0, & \text{if n is even} \\ 1, & \text{if n is odd} \end{cases}$

(v) $1.\alpha_1 \cdot \alpha_2 \cdot \alpha_3 \ldots\ldots\ldots \alpha_{n-1} = \begin{cases} -1, & \text{if } n \text{ is even} \\ 1, & \text{if } n \text{ is odd} \end{cases}$

Note:

(i) $\cos\theta + \cos 2\theta + \cos 3\theta + \ldots + \cos n\theta = \dfrac{\sin(n\theta/2)}{\sin(\theta/2)} \cos\left(\dfrac{n+1}{2}\right)\theta$

(ii) $\sin\theta + \sin 2\theta + \sin 3\theta + \ldots + \sin n\theta = \dfrac{\sin(n\theta/2)}{\sin(\theta/2)} \sin\left(\dfrac{n+1}{2}\right)\theta$.

9. SQUARE ROOT OF COMPLEX NUMBER

Let $x + iy = \sqrt{a+ib}$, Squaring both sides, we get

$(x+iy)^2 = a + ib$

i.e., $x^2 - y^2 = a,\ 2xy = b$

Solving these equations, we get square roots of z.

10. LOCI IN COMPLEX PLANE

(i) $|z-z_0| =$ a represents the circumference of a circle, centred at z_o, radius *a*.

(ii) $|z-z_0| <$ a represents the interior of the circle.

(iii) $|z-z_0| >$ a represents the exterior of this circle.

(iv) $|z-z_1| = |z-z_2|$ represents $\perp$ bisector of segment with endpoints z_1 and z_2.

(v) $\left|\frac{-z_1}{-z_2}\right| = k$ represents: $\begin{Bmatrix} \text{circle, } k \neq 1 \\ \perp \text{ bisector, } k = 1 \end{Bmatrix}$

(vi) $\arg(z) = \theta$ is a ray starting from the origin (excluded) inclined at an $\angle\theta$ with a real axis.

(vii) Circle described on line segment joining z_1 and z_2 as diameter is:

$$(z-z_1)(\bar{z}-\bar{z}_2)+(z-z_2)(\bar{z}-\bar{z}_1)=0$$

(viii) If z_1, z_2, z_3 are the vertices of an equilateral triangle where z_0 is its circumcentre then

(a) $\frac{1}{z_2-z_3}+\frac{1}{z_3-z_1}+\frac{1}{z_1-z_2}=0$

(b) $z_0^1+z_1^1+z_2^1-z_1z_2-z_2z_3-z_3z_1=0$

(c) $z_0^1+z_1^1+z_2^1=3z_/^1$

(ix) If A, B, C, D are four points representing the complex numbers z_1, z_2, z_3, z_4 then

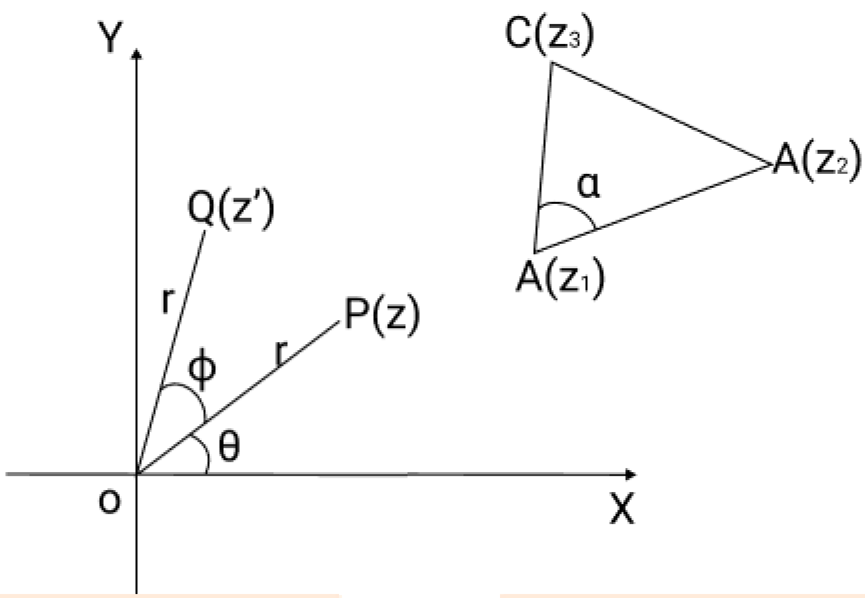
$AB \parallel CD$ if $\frac{Z_4-Z_3}{z_2-z_1}$ is purely real ;

$AB \perp CD$ if $\frac{z_4 - z_3}{z_2 - z_1}$ is purely imaginary

11. VECTORIAL REPRESENTATION OF A COMPLEX

Every complex number can be considered as if it is the position vector of that point. If the point P represents the complex number *z* then,

$\overrightarrow{OP} = z$ and $|\overrightarrow{OP}| = |z|$.



Note:

(i) If $\overrightarrow{OP} = z = re^{i\theta}$ then $\overrightarrow{OQ} = z_1 = re^{i(\theta+\phi)} = z.e^{i\phi}$.

If $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ are of unequal magnitude, then

$OQ = OPe^{i\phi}$

(ii) If z_1, z_2 and z_3 are three vertices of a triangle ABC described in the counterclockwise sense, then

$$\frac{z_3 - z}{z_2 - z} = \frac{AC}{AB}(\cos\alpha + i\sin\alpha) = \frac{AC}{AB}\cdot e^{i\alpha} = \frac{|z_3 - z_1|}{|z_2 - z_1|}\cdot e^{i\alpha}$$

12. SOME IMPORTANT RESULTS

(i) If z_1 and z_2 are two complex numbers, then the distance between z_1 and z_2 is $|z_2 - z_1|$.

(ii) Segment Joining points $A(z_1)$ and $B(z_2)$ is divided by point $P(z)$ in the ratio $m_1 : m_2$ then $z = \frac{m_1 z_2 + m_2 z}{m_1 + m_2}$, m_1 and m_2 are real.

(iii) The equation of the line joining z_1 and z_2 is given by

$$\begin{vmatrix} z & \bar{z} \\ z & \bar{z} \\ z_2 & \bar{z}_2 \end{vmatrix} = 0 \text{ (non parametric form)}$$ Or

$$\frac{z - z}{\bar{z} - \bar{z}} = \frac{z - z_2}{\bar{z} - \bar{z}_2}$$

(iv) $\bar{a}z + a\bar{z} + b = 0$ represents a general form of line.

(v) The general eqn. of circle is:

$z\bar{z} + a\bar{z} + \bar{a}z + b = 0$ (where b is real no.).

Centre : $(-a)$ and radius,

$\sqrt{|a|^2 - b} = \sqrt{a\bar{a} - b}$.

(vi) Circle described on line segment joining z_1 and z_2 as diameter is:

$$(z - z_1)(\bar{z} - \bar{z}_2) + (z - z_2)(\bar{z} - \bar{z}_1) = 0$$

(vii) Four pts. z_1, z_2, z_3, z_4 in anticlockwise order will be concyclic, if and only if

$\theta = \arg\cdot\left(\frac{z_2 - z_4}{z_1 - z_4}\right) = \arg\left(\frac{z_2 - z_3}{z_1 - z_3}\right)$

$\Rightarrow \arg\left(\frac{z_2 - z_4}{z_1 - z_4}\right) - \arg\cdot\left(\frac{z_2 - z_3}{z_1 - z_3}\right) = 2n\pi; (n \in I)$

$\Rightarrow \arg\left[\left(\frac{z_2 - z_4}{z_1 - z_4}\right)\left(\frac{z_1 - z_3}{z_2 - z_3}\right)\right] = 2n\pi$

$\Rightarrow \left(\frac{z_2 - z_4}{z_1 - z_4}\right) \times \left(\frac{z_1 - z_3}{z_2 - z_3}\right)$ is real and positive.

(viii) If z_1, z_2, z_3 are the vertices of an equilateral triangle where z_0 is its circumcentre then

(a) $\frac{1}{z_2 - z_3} + \frac{1}{z_3 - z_1} + \frac{1}{z_1 - z_2} = 0$

(b) $z_0^1 + z_1^1 + z_2^1 - z_1 z_2 - z_2 z_3 - z_3 z_1 = 0$

(c) $z_0^1 + z_1^1 + z_2^1 = 3z^1$

(ix) If A, B, C and D are four points representing the complex numbers z_1, z_2, z_3 and z_4 then

$AB \parallel CD$ if $\frac{z_4 - z_3}{z_2 - z_1}$ is purely real;

$AB \perp CD$ if $\frac{z_4 - z_3}{z_2 - z_1}$ is purely imaginary.

(x) Two points $P(z_1)$ and $Q(z_2)$ lie on the same side or opposite side of the line $\bar{a}z + a\bar{z} + b$ accordingly as $\bar{a}z_1 + a\bar{z}_1 + b$ and $\bar{a}z_2 + a\bar{z}_2 + b$ have same sign or opposite sign.

Important Identities

(i) $x^2 + x + 1 = (x - \omega)(x - \omega^2)$

(ii) $x^2 - x + 1 = (x+\omega)\left(x+\omega^2\right)$

(iii) $x^2 + xy + y^2 = (x - y\omega)\left(x - y\omega^2\right)$

(iv) $x^2 - xy + y^2 = (x + \omega y)\left(x + y\omega^2\right)$

(v) $x^2 + y^2 = (x+iy)(x-iy)$

(vi) $x^3 + y^3 = (x+y)(x+y\omega)\left(x+y\omega^2\right)$

(vii) $x^3 - y^3 = (x-y)(x-y\omega)\left(x-y\omega^2\right)$

(viii) $x^2+y^2+z^2-xy-yz-zx=\left(x+y\omega+z\omega^2\right)\left(x+y\omega^2+z\omega\right)$

or $\left(x\omega+y\omega^2+z\right)\left(x\omega^2+y\omega+z\right)$

or $\left(x\omega+y+z\omega^2\right)\left(x\omega^2+y+z\omega\right)$

(ix) $x^3+y^3+z^3-3xyz=(x+y+z)\left(x+\omega y+\omega^2 z\right)\left(x+\omega^2 y+\omega z\right)$

1. QUADRATIC EXPRESSION

The standard form of a quadratic expression in x is, $f(x)=ax^2+bx+c$, where $a,b,c\in R$ and $a\neq 0$. General form of a quadratic equation in x is, $ax^2+bx+c=0$, where $a,b,c\in R$ and $a\neq 0$.

2. ROOTS OF QUADRATIC EQUATION

(a) The solution of the quadratic equation,

$ax^2+bx+c=0$ is given by $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

The expression $D=b^2-4ac$ is called the discriminant of the quadratic equation.

(b) If α and β are the roots of the quadratic equation $ax^2+bx+c=0$, then

(i) $\alpha+\beta=\frac{-b}{a}$

(ii) $\alpha\beta=\frac{c}{a}$

(iii) $|\alpha-\beta|=\frac{\sqrt{D}}{|a|}$

(c) A quadratic equation whose roots are α and β is $(x-\alpha)(x-\beta)=0$ i.e.,

$x^2-(\alpha+\beta)x+\alpha\beta=0$ i.e.,

$x^2-($ sum of roots $)x+$ product of roots $=0$

Note:

$$y\ =\left(ax^2+bx+c\right)\equiv a\left(x-\alpha\right)\left(x-\beta\right)$$

$$=a\left(x+\frac{b}{2a}\right)^2-\frac{D}{4a}$$

3. NATURE OF ROOTS

(a) Consider the quadratic equation $ax^2+bx+c=0$ where a, $a,b,c\in R$ and $a\neq 0$ then;

(i) $D>0\Leftrightarrow$ roots are real and distinct (unequal).

(ii) $D=0\Leftrightarrow$ roots are real and coincident (equal).

(iii) $D<0\Leftrightarrow$ roots are imaginary.

(iv) If $p+iq$ is one root of a quadratic equation, then the other must be the conjugate $p-iq$ and vice versa. $(p,q\in R \text{ and } i=\sqrt{-1})$.

(b) Consider the quadratic equation $ax^2+bx+c=0$ where $a,b,c\in Q$ and $a\neq 0$ then;

(i) If $D>0$ and is a perfect square, then roots are rational and unequal.

(ii) If $\alpha = p + \sqrt{q}$ is one root in this case, (where p is rational and $\sqrt{q}$ is a surd) then the other root must be the conjugate of it i.e., $\beta = p - \sqrt{q}$ and vice versa.

Note:

Remember that a quadratic equation cannot have three different roots and if it has, it becomes an identity.

4. GRAPH OF QUADRATIC EQUATION

Consider the quadratic expression, $y = ax^2 + bx + c$, $a \neq 0$ and $a, b, c \in R$ then;

(i) The graph between x, y is always a parabola. If $a > 0$ then the shape of the parabola is concave upwards and if $a < 0$ then the shape of the parabola is concave downwards.

(ii) $y > 0 \forall x \in R$, only if $a > 0$ and $D < 0$

(iii) $y < 0 \forall x \in R$, only if $a < 0$ and $D < 0$

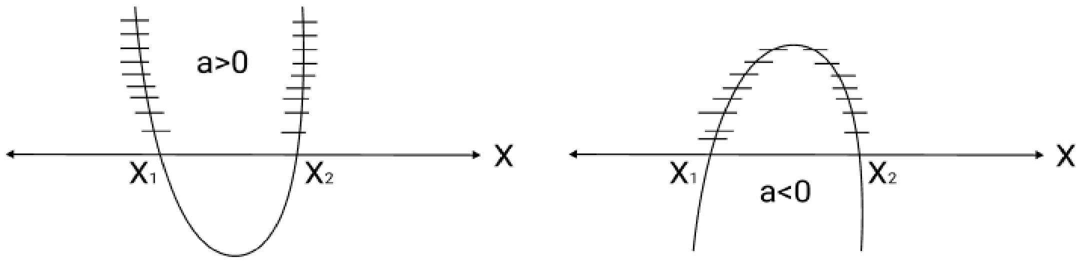
5. SOLUTION OF QUADRATIC INEQUALITIES

$ax^2 + bx + c > 0 (a \neq 0)$

(i) If $D > 0$, then the equation $ax^2 + bx + c = 0$ has two different roots $(x_1 < x_2)$

Then $a > 0 \quad \Rightarrow \quad x \in (-\infty, x_1) \cup (x_2, \infty)$

$a < 0 \quad \Rightarrow \quad x \in (x_1, x_2)$



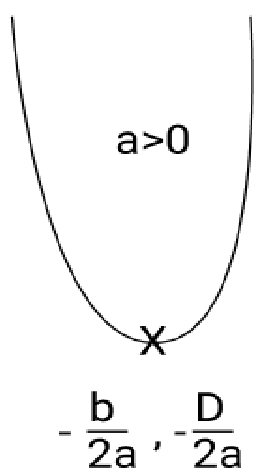
(ii) Inequalities of the form $\frac{P(x)}{Q(x)} \gtrless 0$ can be quickly solved using the method of intervals (wavy curve).

6. MAXIMUM AND MINIMUM VALUE OF QUADRATIC EQUATION

Maximum and minimum value of $y = ax^2 + bx + c$ occurs at $x = -\left(\frac{b}{2a}\right)$ according as:

For $a > 0$, we have:

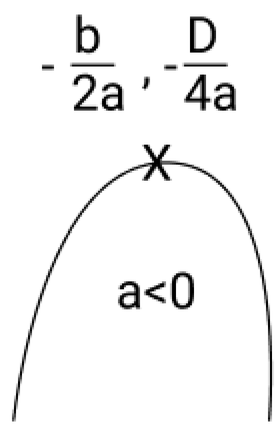
$$y \in \left[\frac{4ac - b^2}{4a}, \infty\right)$$



$y_{\min} = \frac{-D}{4a}$ at $x = \frac{-b}{2a}$, and $y_{\max} \to \infty$

For $a < 0$, we have:

$$y \in \left(-\infty, \frac{4ac - b^2}{4a}\right]$$



$y_{\max} = \frac{-D}{4a}$ at $x = \frac{-b}{2a}$, and $y_{\min} \to \infty$

7. THEORY OF EQUATIONS

If $\alpha_1, \alpha_2, \alpha_3, \ldots\ldots, \alpha_n$ are the roots of the n^{th} degree polynomial equation:

$$f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots\ldots + a_{n-1}x + a_n = 0$$

where $a_0, a_1, \ldots\ldots a_n$ are all real and $a_0 \neq 0$

Then,

$$\sum \alpha_1 = -\frac{a_1}{a_0}$$

$$\sum \alpha_1\alpha_2 = \frac{a_2}{a_0}$$

$\sum \alpha_1\alpha_2\alpha_3 = -\frac{a_3}{a_0}$;

…………

$\alpha_1\alpha_2\alpha_3\ldots\alpha_n = (-1)^n \frac{a_n}{a_0}$

8. LOCATION OF ROOTS

Let $f(x) = ax^2 + bx + c$, where $a > 0$ and $a, b, c \in R$

(i) Conditions for both the roots of $f(x) = 0$ to be greater than a specified number 'k' are:

$D \geqslant 0$ and $f(k) > 0$ and $(-b/2a) > k$

(ii) Conditions for both roots of $f(x) = 0$ to lie on either side of the number 'k' (in other words the number 'k' lies between the roots of $f(x) = 0$ is:

$a\,f(k) < 0$

(iii) Conditions for exactly one root of $f(x) = 0$ to lie in the interval (k_1, k_2) i.e., $k_1 < x < k_2$ are:

$D > 0$ and $f(k_1).f(k_2) < 0$

(iv) Conditions that both the roots of $f(x) = 0$ to be confined between the numbers k_1 and k_2 are $(k_1 < k_2)$:

$D \geqslant 0$ and $f(k_1) > 0$ and $f(k_2) > 0$ and $k_1 < \left(\frac{-b}{2a}\right) < k_2$

9. MAXIMUM AND MINIMUM VALUES OF RATIONAL NUMBERS

Here we shall find the values attained by a rational expresion of the form $\frac{a_1x^2+b_1x+c_1}{a_2x^2+b_2x+c_2}$ for real values of x.

10. COMMON FACTORS

(a) Only One Common Root

Let α be the common root of $ax^2+bx+c=0$ and $a'x^2+b'x+c'=0$, such that $a,a'\neq 0$ and a $b'\neq a'b$. Then, the condition for one common root is:

$$\left(ca'-c'a\right)^2=\left(ab'-a'b\right)\left(bc'-b'c\right)$$

(b) Two common roots

Let α,β be the two common roots of

$ax^2+bx+c=0$ and $a'x^2+b'x+c'=0$ such that $a,a'\neq 0$.

Then, the condition for two common roots is: $\frac{a}{a'}=\frac{b}{b'}=\frac{c}{c'}$

11. RESOLUTION INTO TWO LINEAR FACTORS

The condition that a quadratic function $f(x,y)=ax^2+2hxy+by^2+2gx+2fy+c$ may be resolved into two linear factors is that:

$abc+2fgh-af^2-bg^2-ch^2=0$ or,

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}=0$$

12. FORMATION OF A POLYNOMIAL EQUATION

If $\alpha_1, \alpha_2, \alpha_3, \ldots\ldots, \alpha_n$ are the roots of the n^{th} degree polynomial equation, then the equation is

$$x^n - S_1 x^{n-1} + S_2 x^{n-2} + S_3 x^{n-3} + \ldots\ldots + (-1)^n S_n = 0$$

where S_k denotes the sum of the products of roots taken k at a time.

Particular Cases

(a) Quadratic Equation: If α, β be the roots the quadratic equation, then the equation is :

$$x^2 - S_1 x + S_2 = 0 \quad \text{i.e.} \quad x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

(b) Cubic Equation: If α, β, γ be the roots the cubic equation, then the equation is :

$$x^3 - S_1 x^2 + S_2 x - S_3 = 0 \quad \text{i.e}$$

$$x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma = 0$$

(i) If α is a root of equation $f(x) = 0$, the polynomial $f(x)$ is exactly divisible by $(x - \alpha)$. In other words, $(x - \alpha)$ is a factor of $f(x)$ and conversely.

(ii) Every equation of nth degree $(n \geq 1)$ has exactly n roots \& if the equation has more than n roots, it is an identity.

13. TRANSFORMATION OF EQUATIONS

(i) To obtain an equation whose roots are reciprocals of the roots of a given equation, it is obtained by replacing x by $1/x$ in the given equation.

(ii) Transformation of an equation to another equation whose roots are negative of the roots of a given equation-replace x by $-x$.

(iii) Transformation of an equation to another equation whose roots are square of the roots of a given equation-replace x by $\sqrt{x}$.

(iv) Transformation of an equation to another equation whose roots are cubes of the roots of a given equation-replace x by $x^{1/3}$.