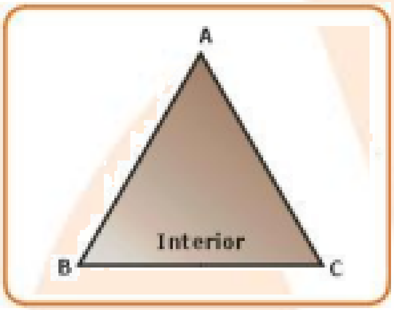
Revision Notes

Class - 9 Maths

Chapter 9 - Areas of Parallelograms and Triangles

Definition of triangle:



A figure in two dimensions, constituting three lines and corners is called a triangle.

Properties of a triangle:

- A triangle is the polygon having least number of sides.
- It is a closed figure in two dimensions formed by **three** line segments and corners thus, it occupies area **bounded** within the sides.

As triangle is the simplest polygon, the area of other polygons can be defined by using area of finite sets of triangles. For example: A hexagon is made up of two triangular parts thus, the area of a hexagon is the union of two triangular regions.

Definition of unit area:

Area enclosed by a figure having sides of unit length is called unit area. It is generally represented as square units and is a positive real number

Notation of area of a polygon:

The area of a polygonal figure A is denoted by $ar(A)$. And in meters, it is denoted by m^2.

Area axioms

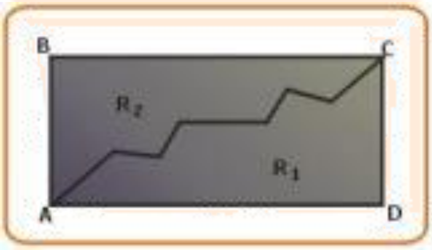
(a) Congruent area axiom:

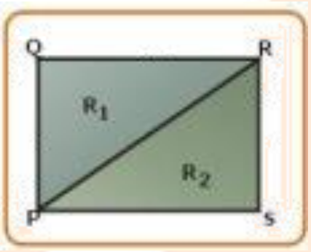
If $\triangle ABC \cong \triangle PQR$ then area of triangle ABC = Area of triangle PQR.

(b) Area monotone axiom:

If R_1 and R_2 two polygonal regions such that $R_1 \subset R_2$ area of $R_1 \leq R_2$.

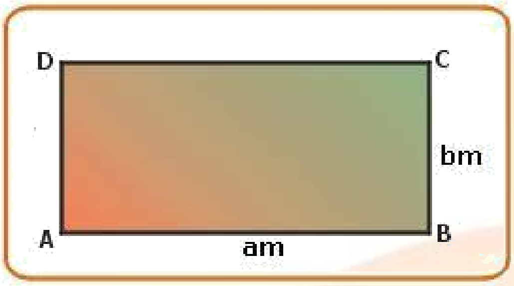
(c) Area addition axiom





If R_1 and R_2 are two polygonal whose intersection is either a finite number of line segments or a single point and $R = R_1 + R_2$ then $ar(R_0) = ar(R_1) + ar(R_2)$. In figs (i) the region is divided into two regions R_1 and R_2.

Area of a rectangular region

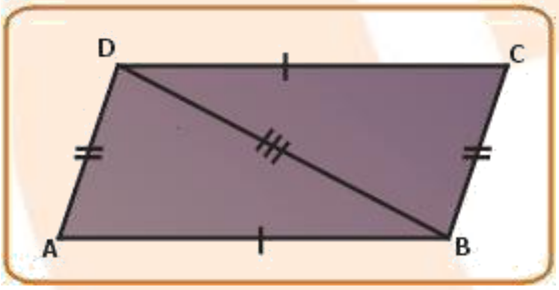


Given that $AB = a$ metres and $AD = b$ metres, hence $ar\left(ABCD\right) = ab$ sq. m. (Using addition area axiom)

Theorem 1

Statement:

Diagonals of a parallelogram divides it into two triangles of equal area.



Given:

ABCD is a parallelogram. AC is one of the diagonals of the parallelogram ABCD.

To prove:

$$ar(ABC) = ar(DBC)$$

Proof:

In triangles ABD and DBC,

$AB = DC$ (Opposite sides of parallelogram)

$AD = BC$ (Opposite sides of parallelogram)

$BD = BD$ (Common side)

(Area congruency axiom)

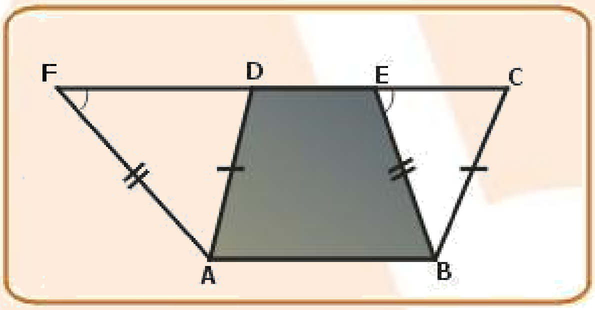
Hence, $ABC \cong DBC$ (SSS congruency)

$ar(ABC) = ar(DBC)$ **(**Using congruent area axiom)

Theorem 2

Statement:

Parallelograms on the same base and between the same parallel lines are equal in area.



Given:

ABCD and ABEF are two parallelograms having same base AB and same parallels AB and CF.

To prove:

Area of parallelogram $ABCD = ABEF$

Proof:

$ar(||^{m}\ ABCD) = ar(ABED) + ar(EBC)$ .... (1) (area addition axiom)

$ar(||^{m}\ ABEF) = ar(ABED) + ar(AFD)$ .... (2) (area addition axiom)

Now in triangles EBC and AFD,

AF = BE (Opposite sides of a parallelogram)

AD = BC (Opposite sides of a parallelogram)

Angle $AFD = BEC$ ($AB \parallel BE$ and FC is a transversal)

Hence are corresponding angles.

$EF = AB = CD$

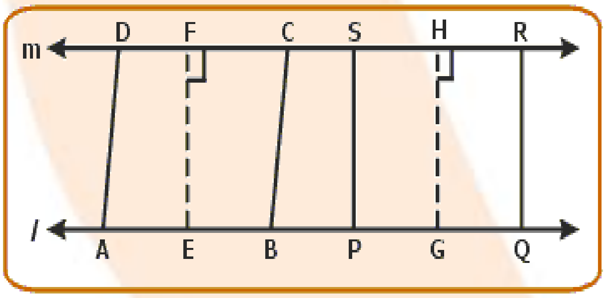
$EF - DE = CD - DE$ i.e., FD = EC

Triangle $EBC \cong AFD$ (SAS congruency condition)

$ar(EBC) = ar(AFD)$ (Area congruency condition)

$ar(||^{m}\ ABCD) = ar(||^{m}\ ABCD)$ From (1), (2) and (3),

Corollary Statement:



Parallelograms on equal bases and between the same parallels are equal in area.

Given: $||^{m}\ ABCD$ and $||^{m}\ PQRS$ are between the same parallels l and m such that $AB = PQ$ (equal bases).

To prove: $ar(||^{m}\ ABCD) = ar(||^{m}\ PQRS)$.

Construction:

Draw the altitude EF and GH.

Proof:

$l \parallel m$ (From given data)

EF = GH (perpendicular distance between the same parallels)

$ar(\|^{m}\ ABCD) = AB \times EF$

$ar(\|^{m}\ PQRS) = PQ \times GH$ (area of a = base x alr)

Since $AB = GH$ (given)

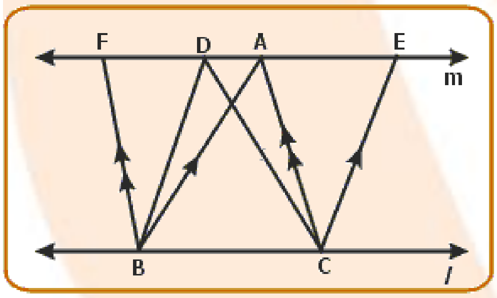
and $EF = GH$(construction)

Hence, $ar(\|^{m}\ ABCD) = ar(\|^{m}\ PQRS)$

Theorem 3

Statement:

Triangles on the same base and between the same parallels are equal in area.



Given:

Triangles ABC and DBC stand on the same BC and between the same parallels l and m.

To prove:

$ar(ABC) = ar(DBC)$

Construction:

CE || AB and BF || CA

Proof:

$||^m$ ABCE and $||^m$ DCBF has same base BC and lies between the same parallels l and m.

$||^m ABCE = ||^m DCBF$ .... (1)

AC is a diagonal of $||^m$ ABCE which divides the parallelogram into two triangles of equal areas.

Similarly, we can prove that

$$ar(BCD) = \frac{1}{2} ar(||^m DCBF)$$

From (1), (2) and (3), we can write

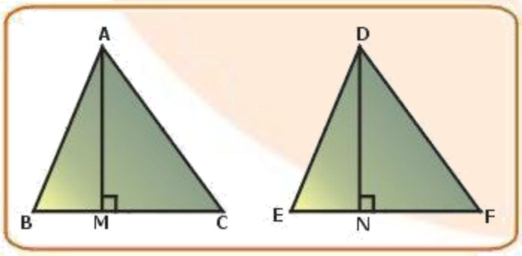
$$ar(ABC) = ar(DBC)$$

Hence the theorem is proved.

Theorem 4

Statement:

Triangles of equal areas, having one side of one of the either triangles equal to one side of the other, have their corresponding altitudes equal.



Given:

Two triangles ABC and DEF are such that:

(i) $ar(ABC) = ar(DEF)$

(ii) $BC = EF$

AM and DN are altitudes of triangle ABC and triangle DEF respectively.

To prove:

$AM = DN$

Proof:

In triangle ABC, AM is the altitude, BC is the base.

$$\Delta ABC = \frac{1}{2} \times BC \times AM$$

In ΔDEF, DN is the altitude and EF is the base.

$$\Delta DEF = \frac{1}{2} \times EF \times DN$$

$$\frac{1}{2} \times BC \times AM = \frac{1}{2} \times EF \times DN$$

Also $BC = EF$ (given)

½ AM = ½ DN

i.e., $AM = DN$.

Hence the theorem is proved.