NCERT Exemplar Solutions for CBSE Class 10

Mathematics

Chapter 7 – Coordinate Geometry

Exercise 7.1

1. The distance of point $P(2,3)$ from the x-axis is

(A) 2

(B) 3

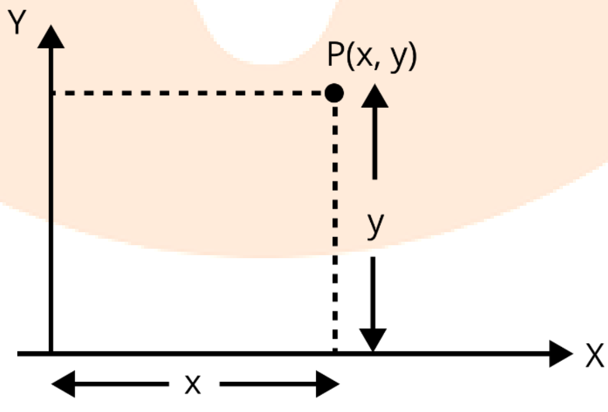
(C) 1

(D) 5

Ans: Correct answer: B.

The perpendicular distance of the point $P(2,3)$ from the $x-$axis will be equal to the y coordinate. So, the distance of the point $P(2,3)$ from the $x-$axis is 3 units.

Hence, the distance of the point $P(2,3)$ from the $x-$axis is 3 units.



2. The distance between the points $A(0,6)$ and $B(0,-2)$ is

(A) 6

(B) 8

(C) 4

(D) 2

Ans: Correct answer: B.

$A(x_1, y_1)$ and $B(x_2, y_2)$ are two points respectively, the distance between them is given as:

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Given points are $A(0,6)$ and $B(0,-2)$.

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$AB = \sqrt{(0-0)^2 + (-2-6)^2}$$

$$AB = 8$$

Hence, the distance between the points $A(0,6)$ and $B(0,-2)$ is 8.

3. The distance of point $P(-6,8)$ from the origin is

(A) 8

(B) $2\sqrt{7}$

(C) 10

(D) 6

Ans: Correct answer: C.

The distance between any two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Given points are $O(0,0)$ and $P(-6,8)$.

$$OP = \sqrt{(-6-0)^2 + (8-0)^2}$$

$$OP = \sqrt{100}$$

$$OP = 10$$

Therefore, the distance of $P(-6,8)$ from origin is 10.

4. The distance between the points $(0,5)$ and $(-5,0)$ is

(A) 5

(B) $5\sqrt{2}$

(C) $2\sqrt{5}$

(D) 10

Ans: Correct answer: B.

Distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ are as follows:

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Given points are $(0,5)$ and $(-5,0)$.

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$AB = \sqrt{(-5-0)^2 + (0-5)^2}$

$AB = \sqrt{50}$

$AB = 5\sqrt{2}$

Hence, the distance between $(0,5)$ and $(-5,0)$ is $5\sqrt{2}$.

5. AOBC is a rectangle whose three vertices are vertices $A(0,3), O(0,0)$ and $B(5,0)$. The length of its diagonal is

(A) 5

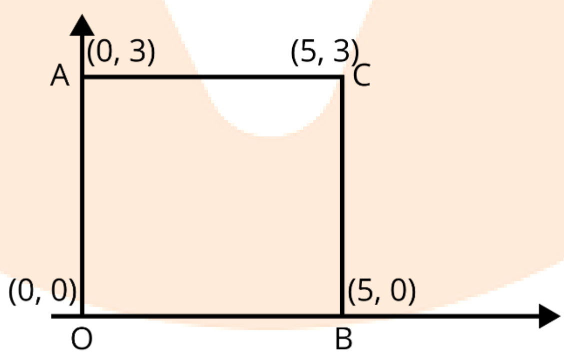
(B) 3

(C) $\sqrt{34}$

(D) 4

Ans: Correct answer: C.

The length of the diagonal of AOBC is AB.



The distance between any two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Given points are $A(0,3), O(0,0)$ and $B(5,0)$.

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$AB = \sqrt{(5-0)^2 + (0-3)^2}$$

$$AB = \sqrt{34}$$

Hence, the length of its diagonal is $\sqrt{34}$.

6. The perimeter of a triangle with vertices $(0,4)$, $(0,0)$ and $(3,0)$ is

(A) 5

(B) 12

(C) **11**

(D) **$7 + \sqrt{5}$**

Ans: Correct answer: B.

Perimeter of triangle $\Delta ABC = AB + BC + AC$.

$(0,4), (0,0)$ and $C(3,0)$ are the three vertices of ΔABC.

Two points $A(x_1, y_1)$ and $B(x_2, y_2)$ are at distance as below

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\Rightarrow AB = \sqrt{(0-0)^2 + (0-4)^2}$$

$\Rightarrow AB = 4$

$\Rightarrow AC = \sqrt{(3-0)^2 + (0-4)^2}$

$\Rightarrow AC = 5$

$\Rightarrow BC = \sqrt{(3-0)^2 + (0-0)^2}$

$\Rightarrow BC = 3$

Perimeter of $\Delta ABC = 4+5+3$

Perimeter of $\Delta ABC = 12cm$

Hence, the perimeter of a triangle with vertices $(0,4),(0,0)$ and $(3,0)$ is 12cm.

7. The area of a triangle with vertices A(3,0), B(7,0) and C(8,4) is

(A) 14

(B) 28

(C) 8

(D) 6

Ans: Correct answer: C.

$A(3,0), B(7,0)$ and $C(8,4)$ are the three vertices of ΔABC.

Area of triangle $\Delta ABC = \frac{1}{2}\left[x_1\left(y_2 - y_3\right) + x_2\left(y_3 - y_1\right) + x_3\left(y_1 - y_2\right)\right]$

Area of triangle $\Delta ABC = \frac{1}{2}\left[3\left(0-4\right) + 7\left(4-0\right) + 8\left(0-0\right)\right]$

$\Delta ABC = 8 sq.units$

Hence, the area of a triangle with vertices $A(3,0), B(7,0)$ and $C(8,4)$ is 8 sq.units.

8. The points $(-4,0),(4,0),(0,3)$ are the vertices of a

(A) Right triangle

(B) Isosceles triangle

(C) Equilateral triangle

(D) Scalene triangle

Ans: Correct answer: B.

Two points $A(x_1, y_1)$ and $B(x_2, y_2)$ are at a distance, which is derived as follows:

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\Rightarrow AB = \sqrt{(4+4)^2 + (0-0)^2}$$

$$\Rightarrow AB = 8$$

$$\Rightarrow AC = \sqrt{(0+4)^2 + (3-0)^2}$$

$$\Rightarrow AC = 5$$

$$\Rightarrow BC = \sqrt{(0-4)^2 + (3-0)^2}$$

$$\Rightarrow BC = 5$$

$AC = BC = 5cm$ and $AB = 8cm.$

Hence, the points $(-4,0),(4,0),(0,3)$ are the vertices of an isosceles triangle.

9. The point which divides the line segment joining the points $(7,-6)$ and $(3,4)$ in ratio 1:2 internally lies in the

(A) I quadrant

(B) II quadrant

(C) III quadrant

(D) IV quadrant

Ans: Correct answer: D.

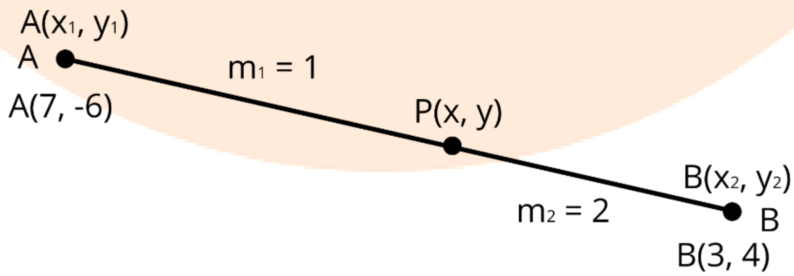
As $x=\frac{m_1x_2+m_2x_1}{m_1+m_2}$ and $y=\frac{m_1y_2+m_2y_1}{m_1+m_2}$

Here, $m_1=1, m_2=2$.

$$\Rightarrow (x,y)=\left(\frac{1(3)+2(7)}{1+2},\frac{1(4)+2(-6)}{1+2}\right)$$

$$\Rightarrow (x,y)=\left(\frac{17}{3},\frac{-8}{3}\right)$$

Hence, we can conclude by saying that the line segment is divided by the line joining $(7,-6)$ and $(3,4)$ in ratio 1:2 internally lies in IV quadrant.



10. The point which lies on the perpendicular bisector of the line segment joining the points A(-2,-5) and B(2,5) is

(A) (0,0)

(B) (0,2)

(C) (2,0)

(D) (-2,0)

Ans: Correct answer: A.

Through the midpoint of AB, the perpendicular bisector of AB passes.

The midpoint of the line joining $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right).$$

$$\text{Mid-point} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$\text{Mid-point} = \left(\frac{-2+2}{2}, \frac{-5+5}{2}\right)$$

$$\text{Mid-point} = (0,0)$$

Hence, the point lying on the perpendicular bisector of the line segment joining the points $A(-2,-5)$ and $B(2,5)$ is $(0,0)$.

11. The fourth vertex D of a parallelogram ABCD whose three vertices are A(-2,3), B(6,7) and C(8,3) is

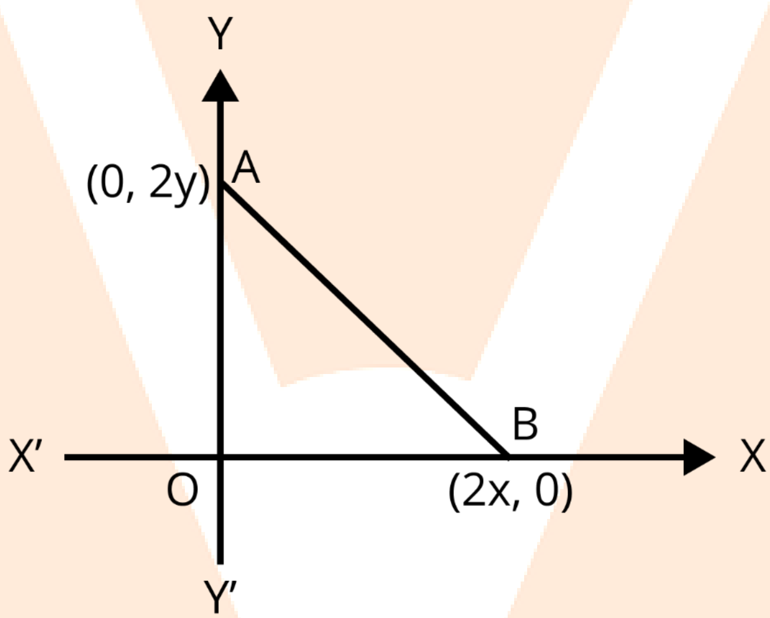
(A) (0,1)

(B) (0,-1)

(C) (-1,0)

(D) (1,0)

Ans: Correct answer: B.



Note that the diagonals AC and BD of parallelogram ABCD bisect each other. So, the midpoint of AC is equal to the midpoint of BD.

The midpoint of $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

$$\Rightarrow \left(\frac{-2+8}{2}, \frac{3+3}{2}\right) = \left(\frac{x_4+6}{2}, \frac{y_4+7}{2}\right)$$

$$\Rightarrow (3,3) = \left(\frac{x_4+6}{2}, \frac{y_4+7}{2}\right)$$

Now, compare both sides:

$\frac{x_4+6}{2}=3$ and $\frac{y_4+7}{2}=3$

$x_4=0$ and $y_4=\ -1$

$(x_4,y_4)=(0,-1)$

Hence, the fourth vertex D of a parallelogram ABCD is $(0,-1)$.

12. If the point $\mathbf{P(2,1)}$ lies on the line segment joining points $\mathbf{A(4,2)}$ and $\mathbf{B(8,4)}$ then

(A) $\mathbf{AP=\frac{1}{3}AB}$

(B) $\mathbf{AP=PB}$

(C) $\mathbf{PB=\frac{1}{3}AB}$

(D) $\mathbf{AP=\frac{1}{2}AB}$

Ans: Correct answer: D.

Let $m_1=k,m_2=1$.

As $x=\frac{m_1x_2+m_2x_1}{m_1+m_2}$ and $y=\frac{m_1y_2+m_2y_1}{m_1+m_2}$,

$\Rightarrow 2=\frac{k(8)+1(4)}{k+1}$ and $1=\frac{k(4)+1(2)}{k+1}$

$\Rightarrow 8k+4=2k+2$ and $4k+2=k+1$

$\Rightarrow k = \frac{-1}{3}$ and $k = \frac{-1}{3}$

So, $\frac{AP}{PB} = \frac{-1}{3}$

$\Rightarrow AP = -1$, which means 1 part outside AB.

And $PB = 3$

$\Rightarrow AP = 1x$ unit and $AB = 3x - 1x = 2x$ units.

So, $AP = \frac{1}{2} AB$.

Hence, $AP = \frac{1}{2} AB$.

13. If P$\left(\frac{a}{3}, 4\right)$ is the midpoint of the line segment joining the points Q(-6,5) and R(-2,3) then the value of a is

(A) - 4

(B) -12

(C) 12

(D) - 6

Ans: Correct answer: B.

$P(x_1, y_1)$ and $Q(x_2, y_2)$ equation's midpoint is given by $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

$$\Rightarrow \left(\frac{a}{3}, 4\right) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$\Rightarrow \left(\frac{a}{3}, 4\right) = \left(\frac{-6-2}{2}, \frac{5+3}{2}\right)$$

$$\Rightarrow \frac{a}{3} = \frac{-8}{2}$$

$$\Rightarrow a = -12$$

Hence, $a = -12$.

14. The perpendicular bisector of the line segment joining the points $A(1,5)$ and $B(4,6)$ cuts the y-axis at

(A) (0,13)

(B) (0,-13)

(C) (0,12)

(D) (13,0)

Ans: Correct answer: A.

The point where the perpendicular bisector of the line segment joining $A(1,5)$ and $B(4,6)$ cuts the y-axis $P(0,y)$.

$$AP = BP$$

$$\Rightarrow AP^2 = BP^2$$

$$\Rightarrow 1 + (y-5)^2 = 16 + (y-6)^2$$

$$\Rightarrow 1 + y^2 - 10y + 25 = 16 + y^2 - 12y + 36$$

$$\Rightarrow y = 13$$

So, the point $P(0, y) = (0,13)$.

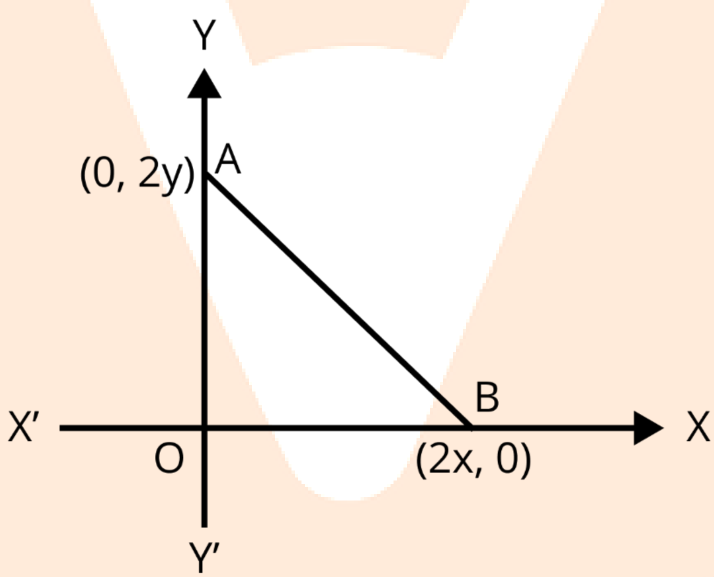
15. The coordinates of the point which is equidistant from the three vertices of the ΔAOB as shown in the Fig. 7.1 is

(A) (x, y)

(B) (y, x)

(C) $\left(\frac{x}{2}, \frac{y}{2}\right)$

(D) $\left(\frac{y}{2}, \frac{x}{2}\right)$



Ans: Correct answer: A.

In any right-angled triangle, the mid-point of the hypotenuse will be equidistant from all the three vertices of the triangle.

The midpoint of $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

Midpoint of $A(0,2y)$ and $B(2x,0)$ is $\left(\frac{2x+0}{2}, \frac{0+2y}{2}\right) = (x, y)$.

Hence, the coordinates of the point which is equidistant from the three vertices of the ΔAOB is (x, y).

16. A circle drawn with origin as the center passes through $\left(\frac{13}{2}, 0\right)$. The point which does not lie in the interior of the circle is

(A) $\left(\frac{-3}{4}, 1\right)$

(B) $\left(2, \frac{7}{3}\right)$

(C) $\left(5, \frac{-1}{2}\right)$

(D) $\left(-6, \frac{5}{2}\right)$

Ans: Correct answer: D.

The distance between any two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by

$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

The radius of the circle is $\sqrt{\left(\frac{13}{2} - 0\right)^2 + (0-0)^2} = 6.5$ units.

Consider the point given in option A,

The distance of $\left(\frac{-3}{4},1\right)$ from origin is $\sqrt{\left(\frac{-3}{4}-0\right)^2+(1-0)^2}=1.25$ units.

As the distance $1.25<6.5$, the point $\left(\frac{-3}{4},1\right)$ lies in the interior of the given circle.

Consider the point given in option B.

The distance of $\left(2,\frac{7}{3}\right)$ from origin is $\sqrt{(2-0)^2+\left(\frac{7}{3}-0\right)^2}=3.0731$ units.

As the distance $3.0731<6.5$, the point $\left(2,\frac{7}{3}\right)$ lies in the interior of the given circle.

Consider the point given in option C,

The distance of $\left(5,\frac{-1}{2}\right)$ from origin is $\sqrt{(5-0)^2+\left(-\frac{1}{2}-0\right)^2}=5.0249$ units.

As the distance $5.0249<6.5$, the point $\left(5,\frac{-1}{2}\right)$ lies in the interior of the given circle.

Consider the point given in option D,

The distance of $\left(-6,\frac{5}{2}\right)$ from origin is $\sqrt{(-6-0)^2+\left(\frac{5}{2}-0\right)^2}=6.5$ units.

As the distance $6.5=6.5$, the point $\left(-6,\frac{5}{2}\right)$ lies on the given circle.

Hence, the point which does not lie in the interior of the circle is $\left(-6,\frac{5}{2}\right)$.

17. A line intersects the y-axis and x-axis at the points P and Q respectively. If $(2,\text{-}5)$ is the midpoint of PQ then the coordinates of P and Q are, respectively

(A) $(0,\text{-}5)$ and $(2,0)$

(B) $(0,10)$ and $(\text{-}4,0)$

(C) $(0,4)$ and $(\text{-}10,0)$

(D) $(0,\text{-}10)$ and $(4,0)$

Ans: Correct answer: D.

As P lies on y-axis, the coordinates of P will be $(0,y)$.

As Q lies on x-axis, the coordinates of Q will be $(x,0)$.

The midpoint of $P(x_1,y_1)$ and $Q(x_2,y_2)$ is given by $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$.

$$\Rightarrow\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=(2,-5)$$

$$\Rightarrow\left(\frac{0+x}{2},\frac{y+0}{2}\right)=(2,-5)$$

$$\Rightarrow x=4 and y=\ -10$$

Hence, the coordinates of P and Q are $(0,-10)$ and $(4,0)$.

18. The area of a triangle with vertices (a,b + c), (b,c + a) and (c,a + b) is

(A) $(a+b+c)^2$

(B) 0

(C) a + b + c

(D) abc

Ans: Correct Answer: B

That area of triangle $\Delta ABC = \frac{1}{2}\left[x_1\left(y_2 - y_3\right) + x_2\left(y_3 - y_1\right) + x_3\left(y_1 - y_2\right)\right]$.

Vertices are (a,b + c), (b,c + a) and (c,a + b).

Area = $\frac{1}{2}\left[a\{c + a - \left(a + b\right)\} + b\{a + b - (b + c)\} + c\{b + c - (c + a)\}\right]$

Area = $\frac{1}{2}\left[a\left(c - b\right) + b(a - c) + c(b - a)\right]$

Area = $\frac{1}{2}\left[ac - ab + ab - bc + bc - ac\right]$

Area = 0

Hence, Area of triangle = 0.

19. If the distance between the points (4,p) and (1,0) is 5 then the value of p is

(A) 4 only

(B) ± 4

(C) - 4 only

(D) 0

Ans: Correct answer B.

The distance between any two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Given $AB = 5$units

$\Rightarrow (AB)^2 = (5)^2$

$\Rightarrow (4-1)^2 + (p-0)^2 = 25$

$\Rightarrow 9 + (p)^2 = 25$

$\Rightarrow (p)^2 = 16$

$\Rightarrow p = \pm 4$

Hence, $p = \pm 4$.

20. If the points A(1,2), O(0,0) and C(a,b) are collinear, then

(A) a = b

(B) a = 2b

(C) 2a = b

(D) a = -b

Ans: Correct answer: C.

Area of triangle $\Delta ABC = \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$.

Area of a triangle is zero if the points are collinear.

$\Rightarrow \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right] = 0$

$$\Rightarrow \frac{1}{2}\left[1(0-b)+0(b-2)+a(2-0)\right]=0$$

$$\Rightarrow \frac{1}{2}(-b+2a)=0$$

$$\Rightarrow \frac{1}{2}(-b+2a)=0$$

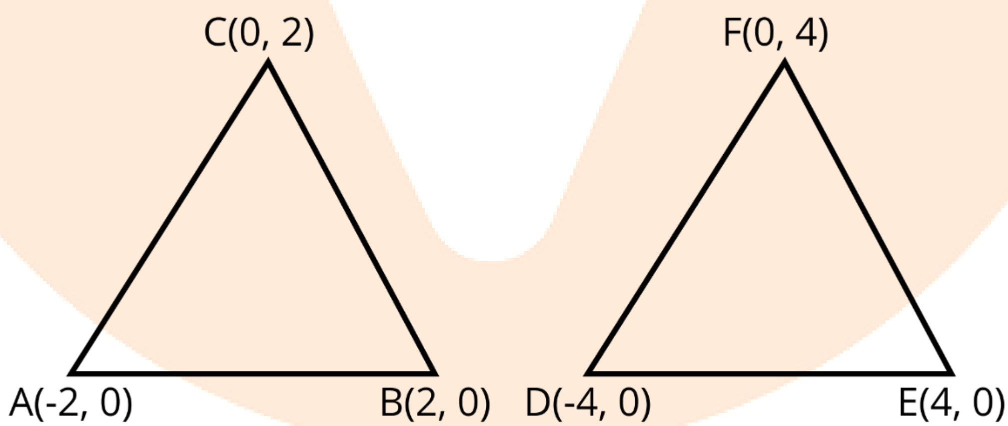
$$\Rightarrow 2a=b$$

Hence, $2a=b.$

Exercise 7.2

1. ΔABC With vertices A(-2,0), B(2,0) and C(0,2) is similar to ΔDEF with vertices D(-4, 0), E(4, 0) and F (0,4)

Ans: The statement is False.



Distance Formula $=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

In ΔABC , $AB=\sqrt{(2+2)^2+(0-0)^2}=\sqrt{(4)^2}=4$

$AC=\sqrt{(0+2)^2+(2-0)^2}=\sqrt{(4+4)}=2\sqrt{2}$

in ΔDEF , $DE=\sqrt{(4+4)^2+(0-0)^2}=\sqrt{(8)^2}=8$

$EF=\sqrt{(0-4)^2+(4-0)^2}=\sqrt{16+16}=\sqrt{32}=4\sqrt{2}$

$DF=\sqrt{(0+4)^2+(4-0)^2}=\sqrt{16+16}=4\sqrt{2}$

Here, $\frac{AB}{DE}=\frac{BC}{EF}=\frac{AC}{DF}$

$\frac{4}{8}=\frac{2\sqrt{2}}{4\sqrt{2}}=\frac{1}{2}$

Hence, ΔABC is similar to ΔDEF .

2. Point P(-4,2) lies on the line segment joining the points A(-4,6) and B(-4,-6)

Ans: The statement is True

The given points $A(-4,6)$ and $B(-4,-6)$

$(x_1,y_1)=(-4,6)$

$(x_2,y_2)=(-4,-6)$

$(x_3,y_3)=(-4,2)$

Point $P(-4,2)$ lie on line AB if the area of triangle $ABP=0$

$\frac{1}{2}[-4(-6-2)-4(2-6)-4(6+6)]$

$\frac{1}{2}[-4(-8)-4(-4)-4(12)]$

$[32+16-48]$
$=0$

$\therefore P(-4,2)$ Must lie on line joining, AB

3. The points $(0,5)$, $(0,\text{-}9)$ and $(3,6)$ are collinear.

Ans: The statement is False

The given points area $(0,5)$, $(0,-9)$ and $(3,6)$

If the point area is collinear then the area of triangle is 0.

$x_1=0, x_2=0, x_3=3$

$y_1=5, y_2=9, y_3=6$

$=\frac{1}{2}[0(-9-6)+0(6-5)+3(5+9)]$

$=\frac{1}{2}[0+0+42]$

$=\frac{42}{2}=21$

Area of triangle $=21$

Here the area of the triangle is not equal to zero.

Hence, the point is not collinear.

4. Point $P(0,2)$ is the point of intersection of y–axis and perpendicular bisector of line segment joining the points $A(-1,1)$ and $B(3,3)$

Ans: The statement is False.

If the point P is a perpendicular bisector of the line joining the point $A(-1,1)$ and $B(3,3)$ then it must be mid-point of AB.

Mid - point of $AB=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$

$(x_1,y_1)=(-1,1)$

$(x_2,y_2)=(3,3)$

It does not represent point P.

Hence, the given statement is false.

5. Points $A(3,1), B(12,-2)$ and $C(0,2)$ cannot be the vertices of a triangle.

Ans: The statement is True

If they are not the vertices of a triangle then

Area of $\Delta ABC=0$

$x_1=3, x_2=12, x_3=0$

$y_1=1, y_2=-2, y_3=2$

Let us find the area of ΔABC

$$=\frac{1}{2}[3(-2-2)+12(2-1)+0(1+2)]$$

$$=\frac{1}{2}[3(-4)+12(1)+0]$$

$= \frac{1}{2}[-12+12]$

$= \frac{0}{2}$

$= 0$

Area of $\Delta ABC = 0$

Hence, they are collinear or not the vertices of a triangle.

6. Points A(4,3), B(6,4), C(5,-6) and D(-3,5) are the vertices of a parallelogram.

Ans: The statement is False

The given points area $A(4,3), B(6,4), C(5,-6), D(-3,5)$

Distance between $AB = \sqrt{(6-4)^2 + (4-3)^2}$

$AB = \sqrt{(2)^2 + (1)^2} = \sqrt{5}$

Distance between $BC = \sqrt{(5-6)^2 + (-6-4)^2}$

$BC = \sqrt{1+100} = \sqrt{101}$

Distance between $CD = \sqrt{(-3-5)^2 + (5+6)^2}$

$CD = \sqrt{64+121} = \sqrt{185}$

Distance between $DA = \sqrt{(4+3)^2 + (3-5)^2}$

$DA = \sqrt{49+4} = \sqrt{53}$

Here, opposite sides are not equal i.e.

$AB \neq CD, BC \neq DA$

Hence, it is not a parallelogram

7. A circle has its Centre at the origin and a point $P(5,0)$ lies on it. The point $Q(6,8)$ lies outside the circle.

Ans: The statement is True.

The center of the circle is $O(0,0)$

If point $P(5,0)$ lies on the circle then the distance between $O(0,0)$ and $P(5,0)$ is the radius of the circle $OP = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$OP = \sqrt{(5-0)^2 + (0-0)^2}$

$OP = \sqrt{(5)^2} = 5$

$OP = 5$

Radius of circle $= 5$

If point $Q(6,8)$ is outside the circle then the distance between $O(0,0)$ and $Q(6,8)$ is greater than the Radius of the circle

$(x_1, y_1) = (0,0)$

$(x_2, y_2) = (6,8)$

$OQ = \sqrt{(6-0)^2 + (8-0)^2}$

$OQ = \sqrt{36 + 64}$

$=\sqrt{100}=10$

Here point OQ is greater than the radius of circle.

Hence, point $Q(6,8)$ lies outside the circle.

8. The point $A(2,7)$ lies on the perpendicular bisector of line segment joining the points $P(6,5)$ and $Q(0,-4)$

Ans: The statement is False.

If point $A(2,7)$ is bisector then it must be mid-point of the line joining the points $P(6,5)$ and $Q(0,-4)$

Mid - point of $PQ=\left(\dfrac{x_1+x_2}{2},\dfrac{y_1+y_2}{2}\right)$

$(x_1,y_1)=(6,5)$

$(x_2,y_2)=(0,-4)$

$=\left(\dfrac{6+0}{2},\dfrac{5-4}{2}\right)=\left(3,\dfrac{1}{2}\right)$

Hence, A does not lies on bisector.

9. Point $C(5,-3)$ is one of the two points of trisection of the line segment joining the points $A(7,-2)$ and $B(1,-5)$

Ans: The statement is True

Let the two point of trisection are C, D

$(x_1, y_1) = (7, -2)$

$(x_2, y_2) = (1, -5)$

$m_1 = 1, m_2 = 2 \ \because$ C divide in ratio 1:2

$$C = \left(\frac{1 \times 1 + 2 \times 7}{1 + 2}, \frac{1 \times (-5) + 2 \times (-2)}{1 + 2} \right)$$

$$C = \left(\frac{1 + 14}{3}, \frac{-5 - 4}{3} \right)$$

$$C = (5, -3)$$

$n_1 = 2, n_2 = 1 \ \because$ C divide in ratio 2:1

$$D = \left(\frac{2 \times 1 + 1 \times 7}{2 + 1}, \frac{2 \times (-5) + 1 \times (-2)}{2 + 1} \right)$$

$$D = \left(\frac{2 + 7}{3}, \frac{-10 - 2}{3} \right)$$

$$D = (3, -4)$$

Hence, the given statement is true.

10. Points A(-6,10), B(-4,6) and C(3,-8) are collinear such that AB = $\frac{2}{9}$AC

Ans: The statement is True.

If the points $A(-6,10), B(-4,6)$ and $C(3,-8)$ are collinear then area of $\Delta ABC = 0$

$$= \frac{1}{2}[-84+72+12]$$

Area of $\Delta ABC = 0$

Hence, A, B and C are collinear

Distance between $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$AB = \sqrt{(-4+6)^2 + (6-10)^2}$$

$$AB = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$$

Distance between $AC = \sqrt{(3+6)^2 + (-8-10)^2}$

$$= \sqrt{81+324}$$

$$= \sqrt{405}$$

$$AC = 9\sqrt{5}$$

Hence, $\frac{AB}{AC} = \frac{2\sqrt{5}}{9\sqrt{5}}$

$$AB = \frac{2}{9}AC$$

11. The point P(-2,4) lies on a circle of radius 6 and Centre C(3,5)

Ans: The statement is False

The radius of the circle is 6 and Centre $C(3,5)$

If point $P(-2,4)$ lies on the circle then the distance between the Centre and point P is equal to the radius of the circle.

Distance between $PC = \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2}$

$\left(x_1, y_1\right) = (-2,4)$

$\left(x_2, y_2\right) = (3,5)$

$PC = \sqrt{(3+2)^2 + (5-4)^2}$

$PC = \sqrt{25+1} = \sqrt{26}$

$PC \neq radius(6)$

Hence, point $P(-2,4)$ not lies on the circle with Centre $C(3,5)$.

12. The points A(- 1, - 2), B(4, 3), C(2, 5) and D(- 3, 0) in that order form a rectangle.

Ans: The statement is True

The given points are A(- 1, - 2),B(4,3),C(2,5) and D(- 3,0)

Length of $AB = \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2}$

$AB = \sqrt{(4+1)^2 + (3+2)^2}$

$AB = \sqrt{50} = 5\sqrt{2}$

Length of $BC = \sqrt{(2-4)^2 + (5-3)^2}$

$BC=\sqrt{4+4}=\sqrt{8}=2\sqrt{2}$

Length of $CD=\sqrt{(-3-2)^2+(0-5)^2}$

$CD=\sqrt{25+25}=\sqrt{50}=5\sqrt{2}$

$DA=\sqrt{4+4}=\sqrt{8}=2\sqrt{2}$

$AB=CD, BC=DA$

Length of $AC=\sqrt{(2+1)^2+(5+2)^2}$

$AC=\sqrt{9+49}=\sqrt{58}$

Length of $BD=\sqrt{(-3-4)^2+(0-3)^2}$

$BD=\sqrt{49+9}=\sqrt{58}$

$AC=BD$ (Diagonals)

Hence, ABCD is a rectangle as

$AB=CD, BC=DA, AC=BD$

Exercise 7.3

1. Name the type of triangle formed by the points A(-5,6), B(-4,-2), C(7,5)

Ans: ABC is a scalene triangle.

Given vertices are $A(-5,6), B(-4,-2), C(7,5)$

Distance formula $=\sqrt{\left(x_2-x_1\right)^2+\left(y_2-y_1\right)^2}$

Distance of $AB=\sqrt{(-4-(-5))^2+(-2-6)^2}$

$=\sqrt{(1)^2+(-8)^2}$

$=\sqrt{1+64}$

$= \sqrt{65}$

$= \sqrt{(11)^2 + (7)^2}$

$= \sqrt{121 + 49}$

$= \sqrt{170}$

Distance of $AC = \sqrt{(7-(-5))^2 + (5-6)^2}$

$= \sqrt{(12)^2 + (-1)^2}$

$= \sqrt{144+1}$

$= \sqrt{145}$

$AB \neq BC \neq AC$

Therefore, ABC is a scalene triangle.

2. Find the points on the x-axis which are at a distance of $2\sqrt{5}$ from the point $(7,-4)$. How many such points are there

Ans: Point are $(9,0)$ and $(5,0)$

Let point on x-axis is $(x,0)$ $\left[\because \text{ y is zero on -axis}\right]$

Given point $(7,-4)$

Distance $= 2\sqrt{5}$

$2\sqrt{5} = \sqrt{(7-x)^2 + (-4,-0)^2}$

Squaring both sides

$(2\sqrt{5})^2 = (7-x)^2 + (-4)^2$

$20 = 49 + x^2 - 14x + 16$

$x^2 - 14x + 65 - 20 = 0$

$x^2 - 14x + 45 = 0$

$x^2 - 9x - 5x + 45 = 0$

$x(x-9) - 5(x-9) = 0$

$(x-9)(x-5) = 0$

$x = 9, x = 5$

Point are $(9,0)$ and $(5,0)$

Hence, two points are there.

3. What type of a quadrilateral do the points $\mathbf{A(2,\text{-}2), B(7,3), C(11,\text{-}1), D(6,\text{-}6)}$ taken in that order, form?

Ans: Rectangle

Let the points be $A(2,-2), B(7,3), C(11,-1), D(6,-6)$ of a quadrilateral ABCD

$AB = \sqrt{(7-2)^2 + (3+2)^2} = \sqrt{5^2 + 5^2} = \sqrt{25+25} = \sqrt{50}$

$BC = \sqrt{(7-11)^2 + (-1-3)^2} = \sqrt{(4)^2 + (-4)^2} = \sqrt{16+16} = \sqrt{32}$

$CD = \sqrt{(6-11)^2 + (-6+1)^2} = \sqrt{(-5)^2 + (-5)^2} = \sqrt{25+25} = \sqrt{50}$

$DA = \sqrt{(6-2)^2 + (-6+2)^2} = \sqrt{(4)^2 + (-4)^2} = \sqrt{16+16} = \sqrt{32}$

$AC = \sqrt{(11-2)^2 + (-1+2)^2} = \sqrt{(9)^2 + (1)^2} = \sqrt{81+1} = \sqrt{82}$

$BD = \sqrt{(6-7)^2 + (-6-3)^2} = \sqrt{(1)^2 + (-9)^2} = \sqrt{1+81} = \sqrt{82}$

$AS, AB = CD$ and $BC = DA$ and $AC = BD$

Hence, the quadrilateral is a rectangle.

4. Find the value of a, if the distance between the points A(-3,-14) and B(a,-5) is 9 units.

Ans: -3

Here, points are $A(-3,-14)$ and $B(a,-5)$

Distance $=9$

$\left(x_1, y_1\right)=(-3,-14)$

$\left(x_2, y_2\right)=(a,-5)$

$9=\sqrt{(a+3)^2+(-5+14)^2}$

Squaring both sides we get

$(9)^2=(a+3)^2+(9)^2$

$(a+3)^2=0$

$\Rightarrow a+3=0$

$a=-3$

Value of a is -3

5. Find a point which is equidistant from the points A(-5,4) and B(-1,6)? How many such points are there?

Ans: Infinite numbers of points are there

Let P (x, y) is a point which is equidistant from point $A(-5,4)$ and $B(-1,6)$ i.e. $PA=PB$

Squaring both sides we get

$PA^2 = PB^2$

$(-5-x)^2 + (4-y)^2 = (-1-x)^2 + (6-y)^2$

$25 + x^2 + 10x + 16 + y^2 - 8y = 1 + x^2 + 2x + 36 + y^2 - 12y$

$\left[\text{Using}: (a+b)^2 = a^2 + b^2 + 2ab; (a-b)^2 = a^2 + b^2 - 2ab\right]$

$25 + 10x + 16 - 8y = 1 + 2x + 36 - 12y$

$10x - 8y + 41 - 2x + 12y - 37 = 0$

$8x + 4y + 4 = 0$

Dividing by 4 we get

$2x + y + 1 = 0 \ldots..(1)$

Mid - point of $AB = \left(\frac{-5-1}{2}, \frac{4+6}{2}\right)$

$= (-3, 5)$

Put point $(-3, 5)$ in eqn. (1)

$2(-3) + 5 + 1$

$-6 + 6$

0

Mid-point of AB satisfy equation (1)

Hence, infinite numbers of points are there.

6. Find the coordinates of the point Q on the x-axis which lies on the perpendicular bisector of the line segment joining the points $\mathbf{A(-5,-2)}$ and $\mathbf{B(4,-2)}$ Name the type of triangle formed by the points Q, A and B.

Ans: ΔABQ Is an isosceles triangle.

Use distance formula $=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Let $Q(x,0)$ t[because On - axis y coordinate is zero]

Q Lies on the perpendicular bisector of the line AB i.e.

$AQ=BQ$

Squaring both sides

$AQ^2=BQ^2$

$(x-5)^2+(0+2)^2=(x-4)^2+(0+2)^2$

$x^2+25-10x+4=x^2+16-8x+4$

$29-10x=20-8x$

$29-20=-8x+10x$

$9=2x$

$\frac{9}{2}=x$

$x=4.5$

$\therefore$ The co-ordinate of Q is $(4.5,0)$

$\because AQ=BQ$ And Q lies on the perpendicular bisector of the line AB

$\therefore \Delta ABQ$ Is an isosceles triangle.

7. Find the value of m if the points $A(5,1), B(-2,-3), C(8,2m)$ are collinear.

Ans: $m = \frac{19}{14}$

Given points are $A(5,1), B(-2,-3), C(8,2m)$

If points are collinear then the area of triangle $= 0$

$\Rightarrow 5(-3-2m) - 2(2m-1) + 8(1+3) = 0$

$\Rightarrow -15 - 10m - 4m + 2 + 32 = 0$

$\Rightarrow -14m + 19 = 0$

$\Rightarrow 14m = 19$

$\Rightarrow m = \frac{19}{14}$

8. If the point $A(2,-4)$ is equidistant from $P(3,8)$ and $Q(-10,y)$, find the values of y. Also find distance PQ.

Ans: $13\sqrt{2}$ *Units* or $\sqrt{290}$ *Units*

Given: Point $A(2,-4)$ is equidistant from $P(3,8)$ and $Q(-10,y)$

$AP = AQ$

Square both sides

$AP^2 = AQ^2$

$(3-2)^2+(8+4)^2=(-10-2)^2+(y+4)^2$ (Using distance formula)

$(1)^2+(12)^2=(12)^2+(y)^2+(4)^2+2\times y\times 4$ $\{Q(a+b)^2=a^2+b^2+2ab\}$

$1+144=144+y^2+16+8y$

$y^2+8y+15=0$

$y^2+3y+5y+15=0$

$y\times(y+3)+5\times(y+3)=0$

$(y+5)(y+3)=0$

$y=-5,\ y=-3$

Case-I when $y=-5$, $PQ=\sqrt{(-10-3)^2+(-5-8)^2}=\sqrt{169+169}=13\sqrt{2}$ Units

Case-II when $y=-3$, $PQ=\sqrt{(-10-3)^2+(-3-8)^2}=\sqrt{169+121}=\sqrt{290}$ Units

9. Find the area of the triangle whose vertices are (-8,4), (-6,6), (-3,9)

Ans: 30 Sq. Units

Given vertices are $(-8,4),(-6,6),(-3,9)$

$x_1=-8x_2=-6x_3=-3$

$y_1=4y_2=6y_3=9$

We know that area of triangle is

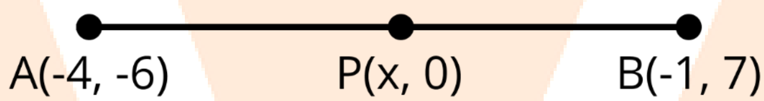
$=\frac{1}{2}[24+30+6]$

$$= \frac{1}{2}[60]$$

$$= 30 Sq.Units$$

10. In what ratio does the x–axis divide the line segment joining the points (-4, -6) and (-1, 7)? Find the coordinates of the point of division.

Ans: Co-ordinate of $P\left(\frac{-34}{13}, 0\right)$



Let the point on x -axis $(x, 0)$ when divides the given points $(-4, -6)$ and $(-1, 7)$ in the Ratio k:1

$$(x_1, y_1) = A(-4, -6)$$

$$(x_2, y_2) = B(-1, 7)$$

$$m_1 = k, m_2 = 1$$

Using section formula, we have $P(x, 0) = \left[\frac{-k-4}{k+1}, \frac{7k-6}{k+1}\right]$

By comparing the left-hand side and the right-hand side we get

$$\frac{7k-6}{k+1} = 0$$

$$7k - 6 = 0$$

$$k = \frac{6}{7}$$

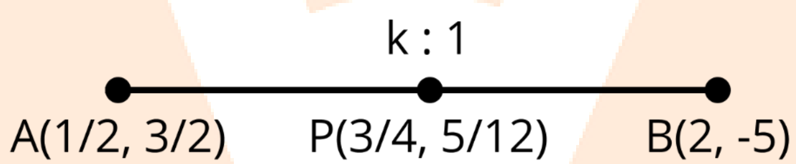
$$x=\frac{-k-4}{k+1}=\frac{-\frac{6}{7}-4}{\frac{6}{7}+1}=\frac{-6-28}{6+7}=\frac{-34}{13}$$

The required ratio is 6:7

Co-ordinate of $P\left(\frac{-34}{13},0\right)$

11. Find the ratio in which the point $P\left(\frac{3}{4},\frac{5}{12}\right)$ divides the line segment joining the points $A\left(\frac{1}{2},\frac{3}{2}\right)$ and B(2,-5).

Ans: 1:5



Let the point $P\left(\frac{3}{4},\frac{5}{12}\right)$ divides the line segment joining the joining the points $A\left(\frac{1}{2},\frac{3}{2}\right)$ and $B(2,-5)$ In the ratio k:1.

$$(x_1,y_1)=\left(\frac{1}{2},\frac{3}{2}\right)$$

$$(x_2,y_2)=(2,-5)$$

$$m_1=k, m_2=1$$

Using section formula, we have

$$\left(\frac{3}{4},\frac{5}{12}\right)=\left[\frac{2k+1/2}{k+1},\frac{-5k+3/2}{k+1}\right]$$

$$\left(\frac{3}{4},\frac{5}{12}\right)=\left[\frac{4k+1}{2k+1},\frac{-10k+3}{2k+2}\right]$$

$$\frac{4k+1}{2k+1}=\frac{3}{4},\frac{-10k+3}{2k+2}=\frac{5}{12}$$

$$16k+4=6k+6-120k+36=10k+10$$

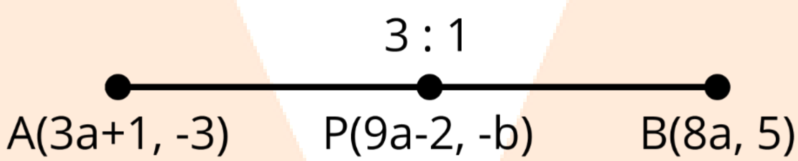
$$16k-6k=6-4-130k=-26$$

$$k=\frac{2}{10}=\frac{1}{5}$$

Therefore the required ratio is 1:5

12. If $P(9a-2,-b)$ divides line segment joining $A(3a+1,-3)$ and $B(8a,5)$ in the ratio 3:1, find the values of a and b.

Ans: $a=1$ and $b=-3$



Point $P(9a-2,-b)$ divides line segment joining the points $A(3a+1,-3)$ and $B(8a,5)$

In ration 3:1

$$\left(x_1,y_1\right)=(3a+1,-3)$$

$$\left(x_2,y_2\right)=(8a,5)$$

$m_1 = 3, m_2 = 1$

Using section formula, we have

$$(9a-2,-b)=\left[\frac{24a+3a+1}{4},\frac{15-3}{4}\right]$$

$$(9a-2,-b)=\left[\frac{27a+1}{4},\frac{12}{4}\right]$$

Equate left-hand side and the right-hand side we get

$$9a-2=\frac{27a+1}{4},-b=\frac{12}{4}$$

$$36a-8=27a+1,-b=3$$

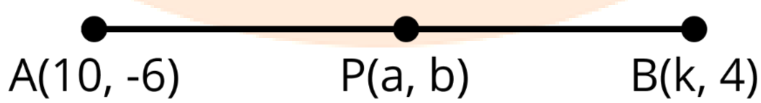
$$9a=9,b=-3$$

$$a=\frac{9}{9}$$

$a=1$ And $b=-3$

13. If P(a,b) is the mid-point of the line segment joining the points A(10,-6) and B(k,4) and a - 2b = 18 , find the value of k and the distance AB.

Ans: $2\sqrt{61}$



Mid - point formula: $x=\frac{x_1+x_2}{2}, y=\frac{y_1+y_2}{2}$

Point $P(a,b)$ divide $A(10,-6)$ and $B(k,4)$ in two equal parts.

$$a=\frac{10+k}{2}, b=\frac{-6+4}{2}=\frac{-2}{2}=-1$$

Given: $a-2b=18$

Put $b=-1$

$a-2(-1)=18$

$a=18-12$

$a=16$

Now, $a=\frac{10+k}{2}$

$16=\frac{10+k}{2}$

$32=10+k$

$32-10=k$

$22=k$

$\therefore A(10,-6), B(22,4)$

$AB=\sqrt{(22-10)^2+(4+6)^2}$

$=\sqrt{(12)^2+(10)^2}$

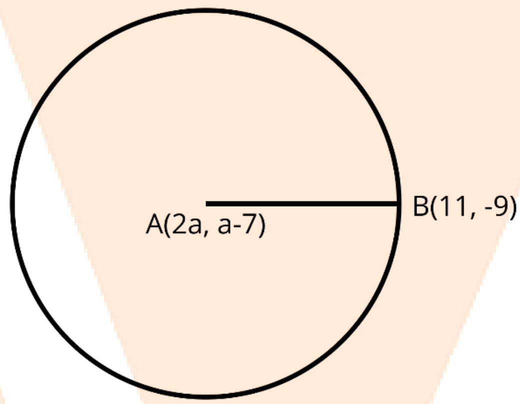
$=\sqrt{144+100}$

$=\sqrt{244}$

$=2\sqrt{61}$

14. The Centre of a circle is $A(2a, a-7)$. Find the values of a if the circle passes through the point $B(11,-9)$ and has diameter $10\sqrt{2}$ units.

Ans: $a = 5,3$



Distance formula $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Given points area $A(2a, a-7)$ and $B(11,-9)$

Diameter $= 10\sqrt{2}$

Radius $= \dfrac{10\sqrt{2}}{2} = 5\sqrt{2}$

Distance $= 5\sqrt{2}$

$(x_1, y_1) = (2a, a-7)$

$(x_2, y_2) = (11,-9)$

$= \sqrt{(11-2a)^2 + (-9-a+7)^2} = (5\sqrt{2})$

Squaring both sides $(11-2a)^2 + (-2-a)^2 = (5\sqrt{2})^2$

$121+4a^2-44a+4+a^2+4a=50$

$502-40a+125-50=0$

$5a^2-40a+75=0$

Dividing by 5 we get

$a^2-8a+15=0$

$a^2-5a-3a+15=0$

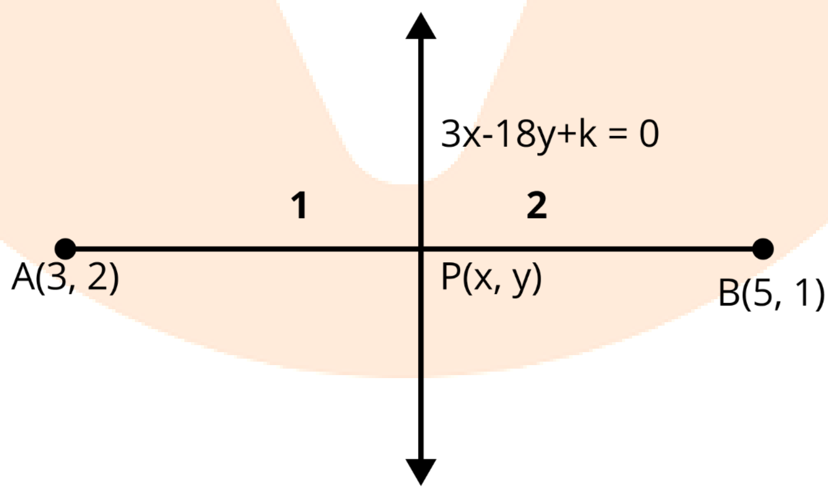
$a(a-5)-3(a-5)=0$

$(a-5)(a-3)=0$

$a=5,3$

15. The line segment joining the points $A(3,2)$ and $B(5,1)$ is divided at the point P in the ratio 1:2 and it lies on the line $3x-18y+k=0$. Find the value of k.

Ans: $k=19$



Here points $A(3,2)$ and $B(5,1)$ is divided at the point P in the ratio 1:2

$(x_1, y_1) = (3,2), (x_2, y_2) = (5,1)$

$m_1 = 1, m_2 = 2$

By section formula we have

$$P(x,y) = \left[\frac{1\times 5 + 2\times 3}{1+2}, \frac{1\times 1 + 2\times 2}{1+2}\right]$$

$$= \left[\frac{5+6}{3}, \frac{1+4}{3}\right]$$

$$= \left[\frac{11}{3}, \frac{5}{3}\right]$$

Line is $3x - 18y + k = 0 \ldots\ldots(1)$

Put point $P\left(\frac{11}{3}, \frac{5}{3}\right)$ in (1)

$$Put x = \frac{11}{3}, y = \frac{5}{3}$$

$$3\left(\frac{11}{3}\right) - 18\left(\frac{5}{3}\right) + k = 0$$

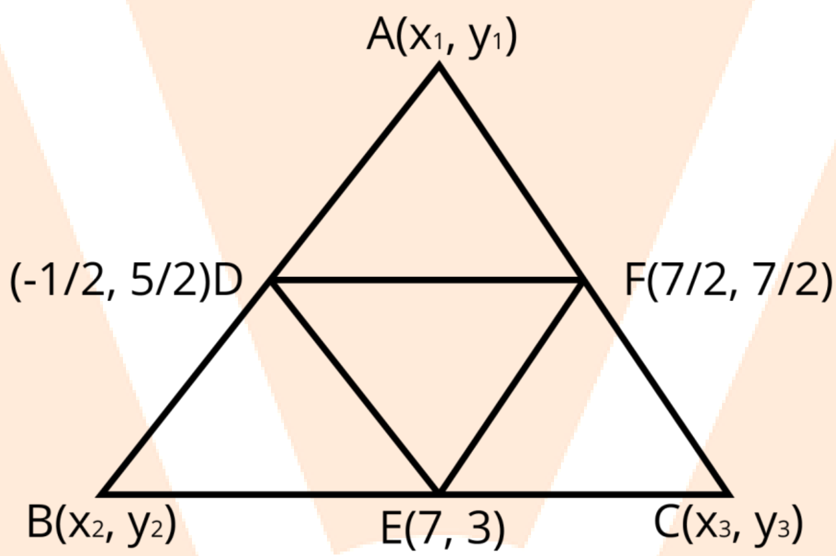
Or, $11 - 30 + k = 0$

Or, $-19 + k = 0$

Or, $k = 19$

16. **If $D\left(\frac{-1}{2},\frac{5}{2}\right), E(7,3)$ and $F\left(\frac{7}{2},\frac{7}{2}\right)$ dare the midpoints of sides of ΔABC find the area of the ΔABC.**

Ans: 11Sq. Units



D Is mid-point of AB using mid-point formula:

$$-\frac{1}{2}=\frac{x_1+x_2}{2},\frac{5}{2}=\frac{y_1+y_2}{2}$$

$x_1+x_2=-1\ldots\ldots(1)$

$5=y_1+y_2\ldots(2)$

E Is mid-point of BC using mid-point formula:

$$\frac{x_2+x_3}{2}=7,\frac{y_2+y_3}{2}=3$$

$x_2+x_3=14\ldots\ldots(3)$

$y_2+y_3=6\ldots.(4)$

F is mid-point of AB using mid-point formula:

$\frac{x_1+x_3}{2}=\frac{7}{2}, \frac{y_1+y_3}{2}=\frac{7}{2}$

$x_1+x_3=7\ldots.(5)$

$y_1+y_3=7\ldots\ldots(6)$

Simplifying the above equations for values of x_1, y_1, x_2, y_2, x_3 and y_3

$x_1+x_2=-1$

$x_3+x_3=14$ {Using (1) and (3)}

$x_1-x_3=-15\ldots..(7)$

Adding equation (5) and (7)

$2x_1=-8$

$x_1=-4$

Put $x_1=4$ in (1) we

$-4+x_2=-1$

$x_2=-1+4=3$

Put $x_2=3$ in (3)

$3+x_3=14$

$x_3=11$

Using equation (4) from equation (4)

$y_1-y_3=--1.....(8)$

Adding equation (6) and (8)

$2y_1=6$

$y_1=3$

Put $y_1 = 3$ in eqn. (2)

$5 - 3 = y_2$

$2 = y_2$

Put $y_2 = 2$ in eqn. (4)

$2 + y_3 = 6$

$y_3 = 6 - 2$

$y_3 = 4$

Hence, $x_1 = -4$ y $1 = 3$

$x_2 = 3 y_2 = 2$

$x_3 = 11 y_3 = 4$

$A = (x_1, y_1) = (-4,3), B = (x_2, y_2) = (3,2), C = (x_3, y_3) = (11,4)$

$= \frac{1}{2}[(-4)(-2) + 3(1) + 11(1)]$

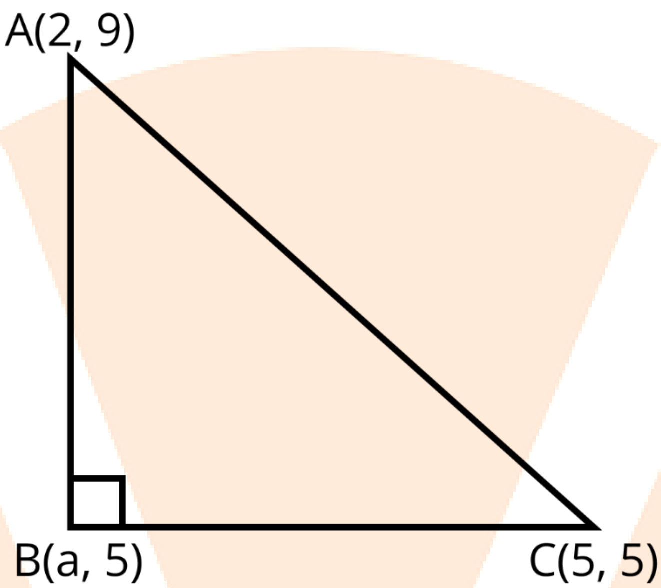
$= \frac{1}{2}[8 + 3 + 11]$

$= \frac{1}{2}[22]$

$= 11 Sq.Units$.

17. The points $A(2,9), B(a,5), C(5,5)$ are the vertices of a ΔABC right angled at B. Find the values of a and hence the area of ΔABC

Ans: 8 Sq. Units



Distance formula $=\sqrt{\left(x_2-x_1\right)^2+\left(y_2-y_1\right)^2}$

ΔABC is a right angle triangle by using Pythagoras theorem

$(AC)^2=(BC)^2+(AB)^2$

$\left[(5-2)^2+(5-9)^2\right]=\left[(2-a)^2+(9-5)^2\right]+\left[(a-5)^2+(5+5)^2\right]$

$(3)^2+(-4)^2=4+a^2-4a+(4)^2+a^2+25-10a$

$9+16=4+a^2-4a+16+a^2+25-10a$

$25=2a^2-14a+45$

$2a^2-14a+45-25=0$

$2a^2-14a+20=0$

Dividing by 2 we have

$a^2-7a+10=0$

$a^2 - 5a - 2a + 10 = 0$

$a(a-5) - 2(a-5) = 0$

$(a-5)(a-2) = 0$

$a = 5, a = 2$

$a = 5$ Is not possible because if $a = 5$ then point B and C coincide.

$\therefore a = 2$

$= \frac{1}{2}[2(5-5) + 2(5-9) + 5(9-5)]$

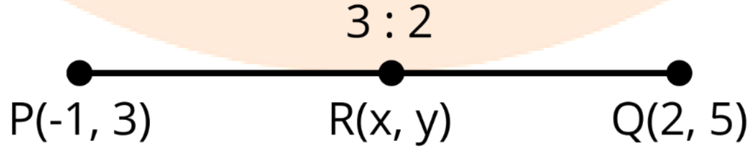
$= \frac{1}{2}[0 - 8 + 20]$

$= \frac{1}{2}[16]$

$= 8 Sq.Units$

18. Find the coordinates of the point R on the line segment joining the points P$(--1,3)$ and Q$(2,5)$ such that PR =$\frac{3}{5}$PQ

Ans: $\left(\frac{4}{5}, \frac{21}{5}\right)$



According to question let $R=(x,y)$ and $PR=\frac{3}{5}PQ$

$$\frac{PQ}{PR}=\frac{3}{5}$$

R Lies on $PQ \therefore\ PQ = PR + RQ$

$$\frac{PR+PQ}{PR}=\frac{5}{3}$$

On dividing separately we get $1+\frac{RQ}{PR}=\frac{5}{3}$

$$\frac{RQ}{PR}=\frac{5}{3}-1=\frac{2}{3}$$

$$\Rightarrow PR:RQ=3:2$$

Hence, R divides PQ in ratio 3: 2 using section formula we have

$$\left(x_1,y_1\right)=(-1,3)$$

$$\left(x_2,y_2\right)=(2,5)$$

$$m_1=3,m_2=2$$

$$R(x,y)=\left(\frac{6-2}{5},\frac{15+6}{5}\right)$$

$$R(x,y)=\left(\frac{4}{5},\frac{21}{5}\right)$$

Here co- ordinates of R is $\left(\frac{4}{5},\frac{21}{5}\right)$

19. Find the values of k if the points $A(k+1,2k)$, $B(3k,2k+3)$ and $C(5k-1,5k)$ are collinear.

Ans: $2,\frac{1}{2}$

If points $A(k+1,2k), B(3k,2k+3)$ and $C(5k-1,5k)$ are collinear then area of triangle is equal to zero

$$[(k+1)(2k+3-5k)+3k(5k-2k)+(5k-1)(2k-2k-3)]=0$$

$$[(k+1)(3-3k)+3k(3k)+5(k-1)(-3)]=0$$

$$\left[3k-3k^2+3-3k+9k^2-15k+3\right]=0$$

$$6k^2-15k+6=0$$

$$6k^2-12k-3k+6=0$$

$$6k(k-2)-3(k-2)=0$$

$$(k-2)(6k-3)$$

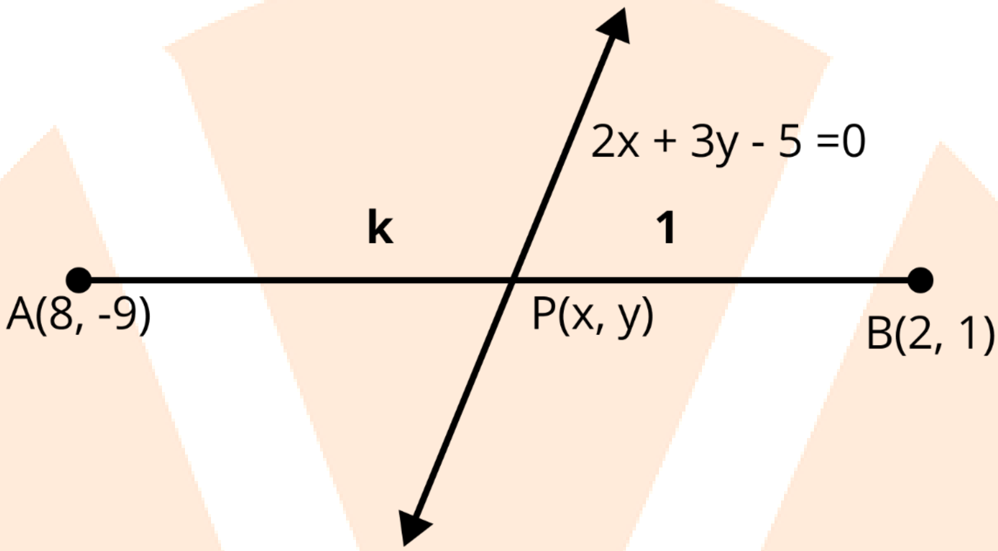
$$k=2, k=\frac{3}{6}$$

$$=\frac{1}{2}$$

Hence, values of k are $2,\frac{1}{2}$

20. Find the ratio in which the line $2x+3y-5=0$ divides the line segment joining the points $A(8,-9)$ and $B(2,1)$. Also find the coordinates of the point of division.

Ans: $P\left(\frac{8}{3},\frac{-1}{9}\right)$



Let point $P(x,y)$ divides the line segment joining the points $A(8,-9)$ and $B(2,1)$ in ratio k:1

$(x_1,y_1)=(8,-9)$

$(x_2,y_2)=(2,1)$

$m_1:m_2=k:1$

Using section formula we have $P(x,y)=\left[\frac{2k+8}{k+1},\frac{k-9}{k+1}\right]$

Given equation is $2x+3y-5=0\ldots(2)$

Put values of x and y in eqn. (2) $2\left[\frac{2k+8}{k+1}\right]+3\left[\frac{k-9}{k+1}\right]-5=0$

$2(2k+3)+3(k-9)-5(k+1)=0$

$4k+16+3k-27-5k-5=0$

$2k-16=0$

$k=8$

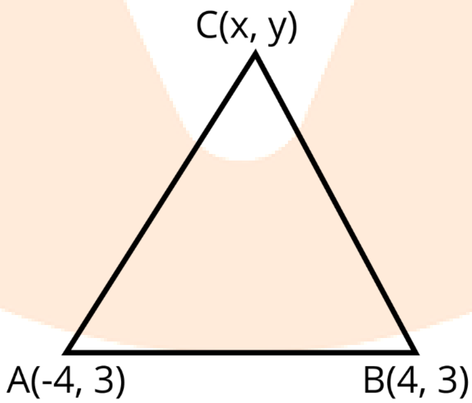
Hence, P divides the line in ration 8:1 put $\mathrm{k}=5$ in eqn. (1)

$(x,y)=\left[\frac{2(8)}{8+1},\frac{8-9}{8+1}\right]$

Required point is $P\left(\frac{8}{3},\frac{-1}{9}\right)$

Exercise 7.4:

1. If (-4,3) and (4,3) are two vertices of an equilateral triangle, find the coordinates of the third vertex, given that the origin lies in the interior of the triangle.



Ans: $\left(0, 3-4\sqrt{3}\right)$

In equilateral triangle $AB = BC = AC$

$$AC = \sqrt{(x+4)^2 + (y-3)^2}$$

$$AC = \sqrt{x^2 + 16 + 8x + y^2 + 9 - 6y}$$

$$\left(\because \ (a+b)^2 = a^2 + b^2 + 2ab \text{and} \left[(a-b)^2 = a^2 + b^2 - 2ab\right]\right)$$

$$BC = \sqrt{(x-4)^2 + (y-3)^2}$$

$$BC = \sqrt{x^2 + 16 - 8x + y^2 + 9 - 6y} \quad \left(\because (a-b)^2 = a^2 + b^2 - 2ab\right)$$

$$\text{AC} = \text{BC}$$

$$\sqrt{x^2 + 16 + 8x + y^2 + 9 - 6y} = \sqrt{x^2 + 16 - 8x + y^2 + 9 - 6y}$$

Squaring both side

$$8x + 8x = 0$$

$$16x = 0$$

$$x = 0$$

$$C = (0, y)$$

Length of $AB = \sqrt{(4+4)^2 + (3-3)^2}$

$$AB = \sqrt{(8)^2} = 8$$

$$AC = AB$$

$$\sqrt{x^2 + 16 + 8x + y^2 + 9 - 6y} = 8$$

Put $x=0$, squaring both side

$$0+16+0+y^2+9-6y=64$$

$$y^2-6y+25-64=0$$

$$y^2-6y-39=0$$

$$y=\frac{6\pm\sqrt{36+156}}{2}$$

$y=\frac{6-8\sqrt{3}}{2}$ (For origin in the interior we take the only term with negative sign)

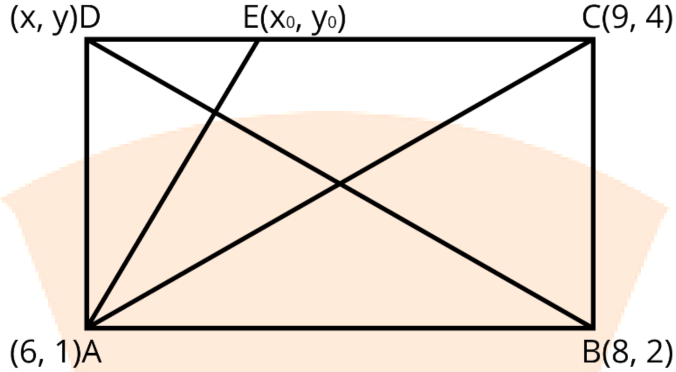
$$y=3-4\sqrt{3}$$

2. A(6,1), B(8,2) and C(9,4) are three vertices of a parallelogram ABCD. If E is the midpoint of DC , find the area of ΔADE.

Ans: $\frac{3}{4} sq\times\ units$

Distance formula $=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

The given points $A(6,1), B(8,2)$ and $C(9,4)$ let $D(x,y)$



As the diagonal of a parallelogram bisect each other.

Here mid-point of AC = mid-point of BD

$$\left(\frac{6+9}{2},\frac{1+4}{2}\right)=\left(\frac{8+x}{2},\frac{2+y}{2}\right)$$

$$\left(\frac{15}{2},\frac{5}{2}\right)=\left(\frac{8+x}{2},\frac{2+y}{2}\right)$$

$$\frac{15}{2}=\frac{8+x}{2}\,\frac{2+y}{2}=\frac{5}{2}$$

$$x=7\,y=3$$

$$D(7,3)$$

E is the mid-point of CD

Let $E\left(x_0,y_0\right)$

$$\left(x_0,y_0\right)=\left(\frac{7+9}{2},\frac{3+4}{2}\right)$$

$$\left(x_0,y_0\right)=\left(\frac{16}{2},\frac{2}{7}\right)$$

$E=\left(8,\frac{7}{2}\right)$

Area of $\Delta ADE=\frac{1}{2}\left[x_1\left(y_2-y_3\right)+x_2\left(y_3-y_1\right)+x_3\left(y_1-y_2\right)\right]$

$=\frac{1}{2}\left[6\left(\frac{7}{2}-3\right)+8(3-1)+7\left(1-\frac{7}{2}\right)\right]$

$=\frac{1}{2}\left[6\times\frac{1}{2}+8\times 2+7\times\frac{-5}{2}\right]$

$=\frac{1}{2}\left[3+16-\frac{35}{2}\right]$

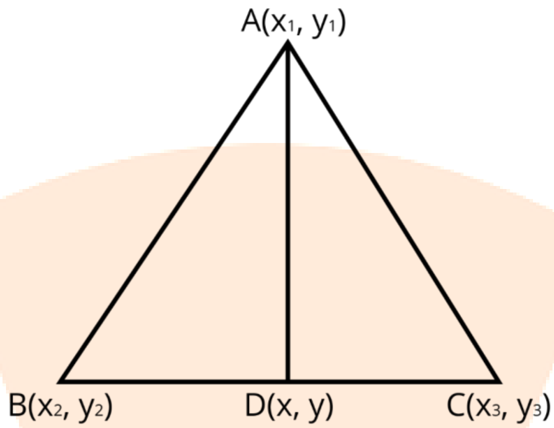
$=\frac{1}{2}\left[\frac{6+32-35}{2}\right]$

Area of $\Delta ADE=\frac{1}{2}\times\frac{3}{2}$

$=\frac{3}{4} squnits$

3. (i) The points $A(x_1,y_1)$, $B(x_2,y_2)\sqrt{a^2+b^2}$ and $C(x_3,y_3)[0,1]$ are the vertices of ΔABC. The median from A meets BC at D. find the coordinates of the point D.

Ans:



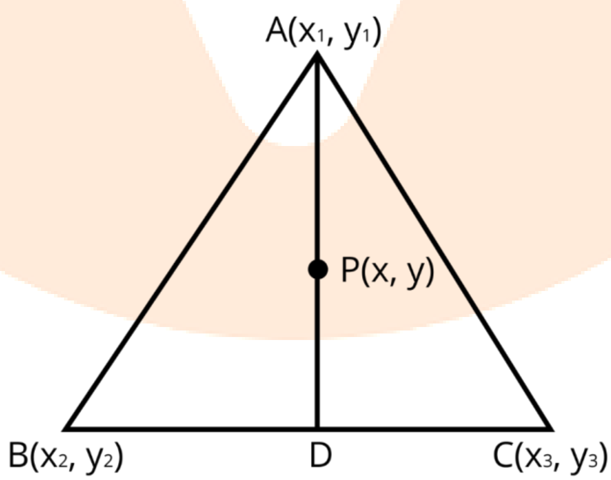
D Is the mid-point of BC

mid - point formula $=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$

Coordinates of $D(x,y)=\left(\frac{x_2+x_3}{2},\frac{y_2+y_3}{2}\right)$ (By midpoint formula)

(ii) The points $A(x_1,y_1)$, $B(x_2,y_2)$ and $C(x_3,y_3)$ are the vertices of ΔABC. Find the coordinates of the point P on AD such that $AP:PD=2:1$

Ans:



Section formula $=\left(\frac{m_1x_2+m_2x_1}{m_1+m_2},\frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$

$D=\left(\frac{x_2+x_3}{2},\frac{y_2+y_3}{2}\right)$ (By Midpoint formula)

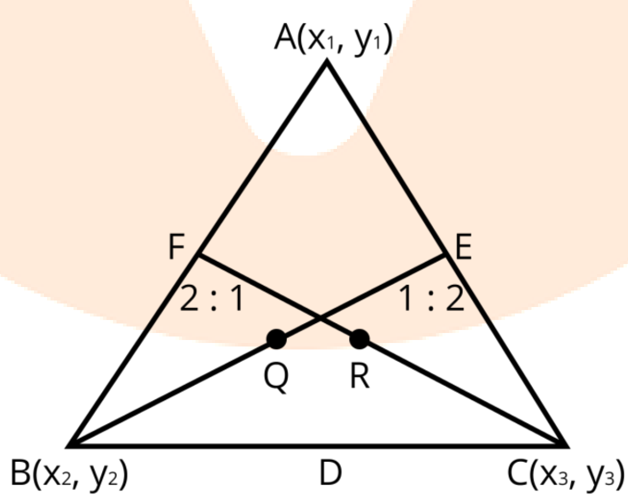
$$P=\left(\frac{m_1x_2+m_2x_1}{m_1+m_2},\frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$$

$$P=\left(\frac{2\times\frac{(x_2+x_3)}{2}+1\times x_1}{2+1},\frac{2\times\frac{(y_2+y_3)}{2}+1\times x_1}{2+1}\right)$$

$$P=\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)$$

(iii) The points $A(x_1,y_1)$, $B(x_2,y_2)$ and $C(x_3,y_3)$ are the vertices of ΔABC. Find the coordinates of points Q and R on medians BE and CF, respectively such that $BQ:QE=2:1$ and $CR:RF=2:1$

Ans:



E is mid-point of AC

$$E=\left(\frac{x_1+x_3}{2},\frac{y_1+y_3}{2}\right)$$

Q divides BF at 2:1

$$Q=\left(\frac{2\times\frac{(x_1+x_3)}{2}+1\times x_2}{2+1},\frac{2\times\frac{(y_1+y_3)}{2}+1\times y_2}{2+1}\right)$$

$$Q=\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)$$

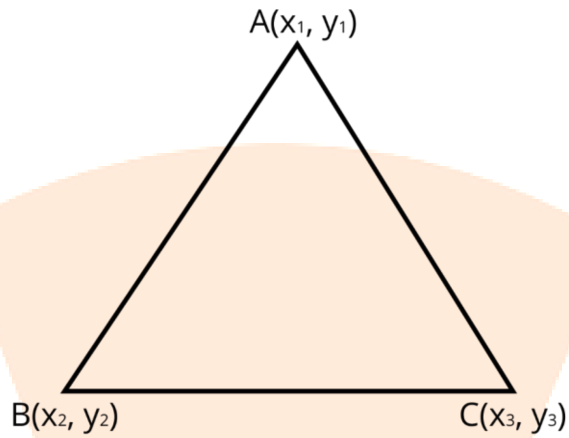
R Divides CF at 2:1

$$R=\left(\frac{2\times\frac{(x_1+x_2)}{2}+1\times x_3}{2+1},\frac{2\times\frac{(y_1+y_2)}{2}+1\times y_3}{2+1}\right)$$

$$R=\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)$$

(iv) The points $A(x_1,y_1), B(x_2,y_2)$ and $C(x_3,y_3)$ are the vertices of ΔABC. what are the coordinates of the centroid of the triangle ABC?

Ans:



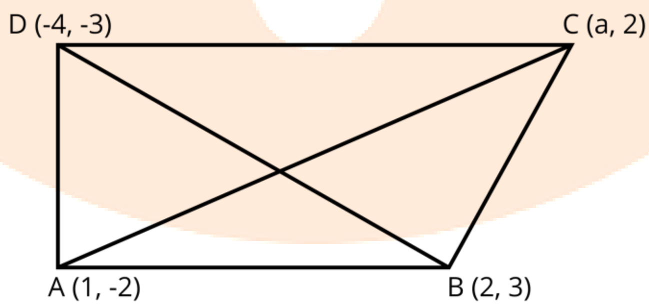
Co-ordinate of centroid

$$= \left(\frac{\text{Sumof all coondinates of all vertices}}{3}, \frac{\text{Sum of all coordinates of all vertices}}{3} \right)$$

Centroid: The centroid is the center point of the triangle which is the intersection of the medians of a Triangle.

$$\Delta \text{ABC Coordinates of centroid } = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$$

4. If the points $A(1,-2), B(2,3), C(a,2)$ and $D(-4,-3)$ form a parallelogram, find the value of a and height of the parallelogram taking AB as base.



Ans: a $= -3$ and height $= \dfrac{24\sqrt{26}}{13}$

We know that diagonals bisect each other

Hence, mid-point of AC $=$ Mid-point of BD

$$\left(\frac{1+a}{2}, \frac{-2+2}{2}\right) = \left(\frac{2-4}{2}, \frac{3-3}{2}\right)$$

$$\left(\frac{1+a}{2}, 0\right) = (-1, 0)$$

$$\frac{1+a}{2} = -1$$

$$1+a = -2$$

$$a = -3$$

$$C(-3, 2)$$

Area of $\Delta \text{ABC} = \frac{1}{2}\left[x_1\left(y_2 - y_3\right) + x_2\left(y_3 - y_1\right) + x_3\left(y_1 - y_2\right)\right]$

$$= \frac{1}{2}[1(3-2) + 2(2+2) + (-3)(-2-3)]$$

$$= \frac{1}{2}[1 + 2(4) + 15]$$

$$= \frac{1}{2}(24) = 12 \text{ sq Units}$$

Area of parallelogram $= 2\times$ Area of ΔABC

Area of parallelogram $= 2\times 12 = 24$ sq Units

Length of $AB = \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2}$

$$= \sqrt{(2-1)^2 + (3+2)^2}$$

$AB = \sqrt{1+25} = \sqrt{26}$ Units

Area of parallelogram = Base × height

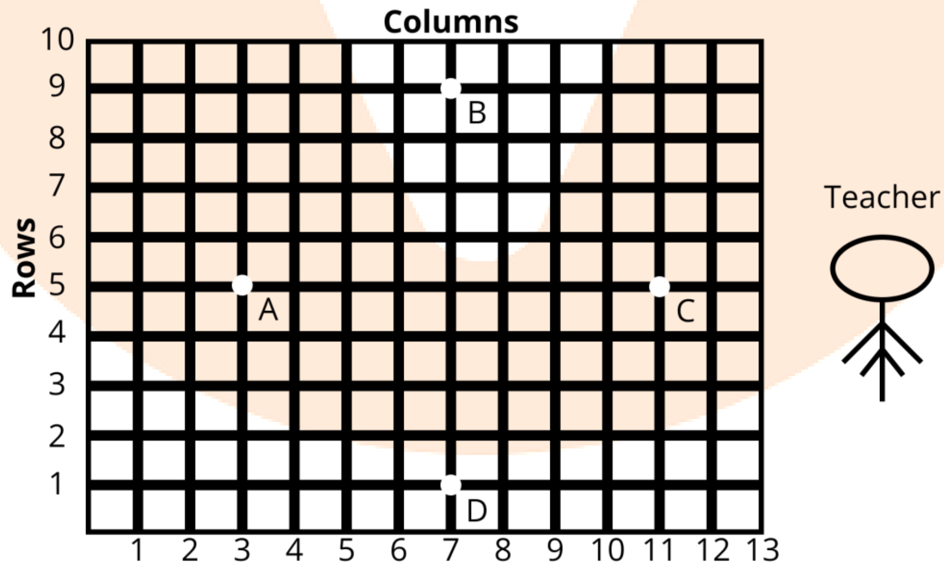
$\frac{24}{\text{Base}} = \text{Height}$

$\text{Height} = \frac{24}{AB}$

$\text{Height} = \frac{24}{\sqrt{26}} \times \frac{\sqrt{26}}{\sqrt{26}}$

$\frac{24\sqrt{26}}{13}$ Units

5. Students of a school are standing in rows and columns in their playground for a drill practice. A, B, C And D are the positions of four students as shown in figure. Is it possible to place Jaspal in the Drill in such a way that he is equidistant from each of the four students A,B,C and D ? If so,what? Should be his position?



Ans: Yes at $AC = (7,5)$

Points are $A(3,5), B(7,9), C(11,5), D(7,1)$

Length of $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$AB = \sqrt{(7-3)^2 + (9-5)^2}$

$AB = \sqrt{16+16}$
$= \sqrt{32} = 4\sqrt{2}$

Length of $BC = \sqrt{(11-7)^2 + (5-9)^2}$

$BC = \sqrt{16+16}$

$= \sqrt{32}$

$= 4\sqrt{2}$

Length of $CD = \sqrt{(7-11)^2 + (1-5)^2}$

$CD = \sqrt{16+16}$

$= \sqrt{32}$

$= 4\sqrt{2}$

Length of $AD = \sqrt{(3-7)^2 + (5-1)^2}$

$AD = \sqrt{16+16}$

$= \sqrt{32}$

$= 4\sqrt{2}$

Length of $AC = \sqrt{(11-3)^2 + (5-5)^2}$

$AC = \sqrt{(8)}$

$= 8$

Length of $\text{BD} = \sqrt{(7-7)^2 + (1-9)^2}$

$\text{BD} = \sqrt{64}$

$= 8$

$\text{AB} = \text{BC} = \text{AD}, \text{AC} = \text{BD}$

Hence, ABCD is square

The diagonals cut each other at mid-point, which is the equidistance from all four corners of square.

Mid – point of $\text{AC} = \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2} \right)$

$(x_1, y_1) = (3,5)$

$(x_2, y_2) = (11,5)$

$\text{AC} = \left(\dfrac{3+11}{2}, \dfrac{5+5}{2} \right)$

$\text{AC} = (7,5)$

This should be the position of Jaspal.

6. Ayush starts walking from his house to the office. Instead of going to the office directly, he goes to a bank first, from there to his daughter's school and then reaches the office. What is the extra distance travelled by Ayush in reaching his office? (Assume that all distances covered are in straight lines). If

the house is situated at (2,4) , bank at (5,8) , school at (13,14) and office at (13, 26) and coordinates are in km.

Ans: The given point are $(2,4),(5,8),(13,14),(13,26)$

Distance between house and bank

$$=\sqrt{\left(x_2-x_1\right)^2+\left(y_2-y_1\right)^2}$$

$$=\sqrt{(5-2)^2+(8-4)^2}$$

$$=\sqrt{9+16}=5 \text{ Km}$$

Distance between bank and school

$$=\sqrt{(13-5)^2+(14-8)^2}$$

$$=\sqrt{64+36}$$

$$=\sqrt{100}=10 \text{ Km}$$

Distance between school and office

$$=\sqrt{(13-13)^2+(26-14)^2}$$

$$=\sqrt{(12)^2}=12 \text{ Km}$$

Distance between office and house

$$=\sqrt{(13-2)^2+(26-14)^2}$$

$$=\sqrt{121+484}$$

$$=\sqrt{605}$$

$$=24\cdot 59 \text{ Km}$$

Total distance covered from house to bank, bank to school, school to office $=5+10+12=27$

Extra distance covered $=27-24.59=2.41\text{km}$.