Important Questions for Class 11

Mathematics

Chapter 3 – Trigonometric Functions

Very Short Answer Questions **1 Mark**

1. Find the radian measure corresponding to $5°\ 37'\ 30''$

Ans: Converting the given value to a pure degree form

$$5^\circ 37'30'' = 5^\circ 37'\left(\frac{30}{60}\right)'$$

$$\Rightarrow 5^\circ 37'60'' = 5^\circ\left(\frac{75}{2}\right)'$$

$$\Rightarrow 5^\circ 37'60'' = 5^\circ\left(\frac{75}{2(60)}\right)^\circ$$

$$\Rightarrow 5^\circ 37'60'' = \left(\frac{45}{8}\right)^\circ$$

Degree to Radian Conversion

$$\left(\frac{45}{8}\right)\left(\frac{\pi}{180}\right) = \frac{\pi}{32}\text{rad}$$

2. Find degree measure corresponding to $\left(\frac{\pi}{16}\right)^c$

Ans: Converting the given value from radian to degree form

$$\frac{\pi}{16} \times \frac{180}{\pi} = \left(\frac{45}{4}\right)^\circ$$

Simplify degree form

$$\left(\frac{45}{4}\right)^\circ = 11^\circ 15'$$

3. Find the length of an arc of a circle of radius 5cm subtending a central angle measuring $15°$

Ans: The arc of a circle with a radius of 5cm with a central angle of 15° should be of the length $\frac{5\pi}{12}\text{cm}$ using the formula $\text{Arc}=\pi\times(\theta)$.

4. Find the value of $\frac{19\pi}{3}$

Ans: We have $\tan\frac{19\pi}{3}$

$$\tan\frac{19\pi}{3}=\tan\left(6\frac{\pi}{3}\right)$$

$$=\tan\left(6\pi+\frac{\pi}{3}\right)$$

$$=\tan\left(3\times2\pi+\frac{\pi}{3}\right)$$

$$=\tan\left(\frac{\pi}{3}\right)$$

$$=\sqrt{3}$$

5. Find the value of $\sin(-1125^{\circ})$

Ans: We have $\sin(-1125^{\circ})$

$$\sin\left(-\frac{1125}{360}\times360^{\circ}\right)$$

$$=-\sin\left(\left(3+\frac{45}{360}\right)\times360^{\circ}\right)$$

$$=-\sin(45^{\circ})$$

$$=-\frac{1}{\sqrt{2}}$$

6. Find the value of $\tan(15^{\circ})$

Ans: We have $\tan15^{\circ}$

$$\tan15^{\circ}=\tan(60^{\circ}-45^{\circ})$$

$$=\frac{\tan 60^\circ-\tan 45^\circ}{1+\tan 60^\circ\times\tan 45^\circ}$$
$$=\frac{\sqrt{3}-1}{\sqrt{3}+1}$$

7. If $\sin A=\frac{3}{5}$ and $\frac{\pi}{2}<A<$ find $\cos A$

Ans: The condition $\frac{\pi}{2}<A$ denotes that we need to take account for second quadrant, hence the cosine value will be negative.
Therefore,
$$\cos A=\frac{-4}{5}$$

8. If $\tan A=\frac{a}{a+1}$ and $\tan B=\frac{1}{2a+1}$ then find the value of $A+B$

Ans: $\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$
$$=\frac{\frac{a}{a+1}+\frac{1}{2a+1}}{1-\frac{a}{a+1}\cdot\frac{1}{2a+1}}$$
$$=\frac{\frac{2a^2+2a+1}{(a+1)(2a+1)}}{\frac{(a+1)(2a+1)-a}{(a+1)(2a+1)}}$$
$$=1$$
Which can only be possible if $A+B=45^\circ$.

9. Express $\sin 12\theta+\sin 4\theta$ as the product of sines and cosine
Ans: Using the trigonometric difference formula, we get
$$\sin 12\theta+\sin 4\theta=\sin(8\theta+4\theta)+\sin(8\theta-4\theta)$$
$$=2\sin 8\theta\cos 4\theta$$

10. Express $2\cos 4x \sin 2x$ as an algebraic sum of sines or cosine.

Ans: $2\cos 4x\sin 2x = \sin(2x+4x)+\sin(2x-4x)$

$=\sin 6x+\sin(-2x)$

$=\sin 6x-\sin 2x$

11. Write the range of $\cos\theta$

Ans: The cosine function is a periodic function with a domain of $\mathbb{R}$ and a range of $[-1,1]$.

12. What is domain of $\sec\theta$

Ans: The **secant** function is the **reciprocal of the cosine function**, it has a domain of $\mathbb{R}-\left\{(2n+1)\dfrac{\pi}{2};n\in\mathbb{Z}\right\}$ because those are the points where the cosine function equates to 0.

13. Find the principal solution of $\cot x = 3$

Ans: The principal solution of $\cot x = 3$ is for the following input values $x=\dfrac{5\pi}{6},\dfrac{11\pi}{6}$.

14. Write the general solution of $\cos\theta = 0$

Ans: The general solution for the equation $\cos\theta=0$ is $\theta=(2n+1)\dfrac{\pi}{2}, n\in\mathbb{Z}$.

15. If $\sin x=\dfrac{\sqrt{5}}{3}$ and $0 < x < \dfrac{\pi}{2}$ find the value of $\cos 2x$

Ans: We know that $\cos 2x = 1-\sin^2 x$

$\cos 2x = 1-2\left(\dfrac{\sqrt{5}}{3}\right)^2$

$=1-2\times\dfrac{5}{9}$

$=-\dfrac{1}{9}$

16. If $\cos x = -\frac{1}{3}$ and x lies in quadrant III, find the value of $\sin\frac{x}{2}$

Ans: We know that $\cos 2x = 1 - 2\sin^2 x$

$$\cos\left(2\left(\frac{x}{2}\right)\right) = 1 - 2\sin^2\left(\frac{x}{2}\right)$$

$$\Rightarrow -\frac{1}{3} = 1 - 2\sin^2\frac{x}{2}$$

$$\Rightarrow 2\sin^2\frac{x}{2} = 1 + \frac{1}{3}$$

$$\Rightarrow \sin^2\frac{x}{2} = \frac{2}{3}$$

$$\Rightarrow \sin\frac{x}{2} = \pm\sqrt{\frac{2}{3}}$$

$$\Rightarrow \sin\frac{x}{2} = \sqrt{\frac{2}{3}} \quad [\text{2nd Quadrant}]$$

17. Convert into radian measures $-47°30'$

Ans: Convert into pure degree form and then convert to radian

$$-47°30' = -\left(47 + \frac{30}{60}\right)^{\circ}$$

$$= -\left(47 + \frac{1}{2}\right)^{\circ}$$

$$= -\left(\frac{95}{2} \times \frac{\pi}{180}\right)\text{rad}$$

$$= -\frac{19\pi}{72}\text{rad}$$

18. Evaluate $\tan 75°$

Ans: Use the trigonometric addition formula for the tangent function

$$\tan 75^{\circ} = \tan(45^{\circ} + 30^{\circ})$$

$$= \frac{\tan 45^{\circ} + \tan 30^{\circ}}{1 - \tan 45^{\circ}\tan 30^{\circ}}$$

$$=\frac{\sqrt{3}+1}{\sqrt{3}-1}$$

19. Prove that $\sin(40+\theta)\cdot\cos(10+\theta)-\cos(40+\theta)\cdot\sin(10+\theta)=\frac{1}{2}$

Ans: Let us take the left-hand side of the equation and make some manipulations.
We know, $\sin(a-b)=\sin a\cos b-\cos a\sin b$

$$\text{L.H.S}=\sin(40+\theta)\cos(10+\theta)-\cos(40+\theta)\sin(10+\theta)$$

$$=\sin[40+\theta-10-\theta]=\sin 30$$

$$=\frac{1}{2}$$

20. Find the principal solution of the eq. $\sin x=\frac{\sqrt{3}}{2}$

Ans: The principal solution of $\sin x=\frac{\sqrt{3}}{2}$ is the input values of $x=\frac{\pi}{3},\frac{2\pi}{3}$

21. Prove that $\cos\left(\frac{\pi}{4}+x\right)+\cos\left(\frac{\pi}{4}-x\right)=\sqrt{2}\cos x$

Ans: Let us start with the left-hand side and use the trigonometric differences formula for the cosine function

$$\text{L.H.S}=\cos\left(\frac{\pi}{4}+x\right)+\cos\left(\frac{\pi}{4}-x\right)$$

$$=2\cos\frac{\pi}{4}\cos x$$

$$=2\left(\frac{1}{\sqrt{2}}\right)\cos x$$

$$=\sqrt{2}\cos x$$

$$=\text{R.H.S}$$

22. Convert into radian measures $-37°30'$

Ans: Convert into pure degree form and then convert from degree to radian

$$-37°30' = \left(37 + \frac{30}{60}\right)^{\circ}$$

$$= -\left(\frac{75}{2}\right)^{\circ}$$

$$= -\frac{75}{2} \times \frac{\pi}{180} \text{rad}$$

$$= -\frac{5\pi}{24} \text{rad}$$

23. Prove

$$\mathbf{Sin\ (n+1)\ x\ Sin\ (n+2)\ x\ +\ Cos\ (n+1)\ x.\ Cos\ (n+2)\ x\ =\ Cos\ x}$$

Ans: $\text{L.H.S.} = \sin(n+1)x\sin(n+2)x + \cos(n+1)x\cos(n+2)x$

$$= \cos\{(n+1)x - (n+2)x\}$$

$$= \cos(nx + x - n - 2x)$$

$$= \cos(-x)$$

$$= \cos(x)$$

24. Find the value of $\mathbf{Sin\frac{31\pi}{3}}$

Ans: We have $\sin\frac{31\pi}{3}$

$$\text{Sin}\frac{31\pi}{3} = \text{Sin}\left(10\pi + \frac{\pi}{3}\right)$$

$$= \text{Sin}\left(2\pi \times 5 + \frac{\pi}{3}\right) \quad [\text{Periodic Function}]$$

$$= \text{Sin}\frac{\pi}{3}$$

$= \frac{\sqrt{3}}{2}$

25. Find the principal solution of the eq. $\tan x = -\frac{1}{\sqrt{3}}$.

Ans: The principal solution of the equation $\tan x = -\frac{1}{\sqrt{3}}$ will be the input values of $x = \frac{5\pi}{6}, \frac{11\pi}{6}$

26. Convert into radian measures $5°\ 37'\ 30''$

Ans: Converting the given value to a pure degree form

$5^{\circ}37'30'' = 5^{\circ}37'\left(\frac{30}{60}\right)'$

$\Rightarrow 5^{\circ}37'60'' = 5^{\circ}\left(\frac{75}{2}\right)'$

$\Rightarrow 5^{\circ}37'60'' = 5^{\circ}\left(\frac{75}{2(60)}\right)^{\circ}$

$\Rightarrow 5^{\circ}37'60'' = \left(\frac{45}{8}\right)^{\circ}$

Degree to Radian Conversion

$\left(\frac{45}{8}\right)\left(\frac{\pi}{180}\right) = \frac{\pi}{32}\text{rad}$

27. Prove $\text{Cos}70°.\ \text{Cos}10° +\ \text{Sin}70°.\ \text{Sin}10° = \frac{1}{2}$

Ans: Starting with the left-hand side and using the trigonometric differences formula for the **cosine function**.

$\text{L.H.S} = \cos\left(70^{\circ} - 10^{\circ}\right)$

$= \cos 60^{\circ}$

$= \frac{1}{2}$

28. Evaluate $2\text{Sin}\frac{\pi}{12}$

Ans: Use the trigonometric difference formula for the sine function and expand

$$2\sin\frac{\pi}{12} = 2\sin\left[\frac{\pi}{4} - \frac{\pi}{6}\right]$$

$$= 2\left[\sin\frac{\pi}{4}\cos\frac{\pi}{6} - \cos\frac{\pi}{4}\sin\frac{\pi}{6}\right]$$

$$= 2\left[\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}\right]$$

$$= \frac{\sqrt{3}-1}{\sqrt{2}}$$

29. Find the solution of $\text{Sin}\,x = -\frac{\sqrt{3}}{2}$

Ans: We are required to find the general solution for the equation $\sin x = -\frac{\sqrt{3}}{2}$

$$\text{Sin}\,x = -\frac{\sqrt{3}}{2}$$

$$\Rightarrow \text{Sin}\,x = \text{Sin}\left(\pi + \frac{\pi}{3}\right)$$

$$\Rightarrow \text{Sin}\,x = \text{Sin}\frac{4\pi}{3}$$

When

$$\text{Sin}\,\theta = \text{Sin}\,\alpha$$

$$\theta = n\pi + (-1)^n \cdot \alpha$$

$$x = n\pi + (-1)^n \cdot \frac{4\pi}{3}$$

30. Prove that $\frac{\text{Cos}\,9° - \text{Sin}\,9°}{\text{Cos}\,9° + \text{Sin}\,9°} = \tan 36°$

Ans: Let us start with the right-hand side and use the trigonometric differences formula for the tangent function.

$$\text{R.H.S} = \tan 36°$$
$$= \tan\left(45^\circ - 9^\circ\right)$$
$$= \frac{\tan 45^\circ - \tan 9^\circ}{1 + \tan 45^\circ \tan 9^\circ}$$
$$= \frac{1 - \tan 9^\circ}{1 + \tan 9^\circ}$$
$$= \frac{\cos 9^\circ - \sin 9^\circ}{\cos 9^\circ + \sin 9^\circ}$$
$$= \text{L.H.S.}$$

31. Find the value of $\tan\frac{19\pi}{3}$

Ans: We have $\tan\left(\frac{19\pi}{3}\right)$

$$\tan\frac{19\pi}{3} = \tan\left(6\pi - \frac{\pi}{3}\right)$$
$$= \tan\left[3\times 2\pi + \frac{\pi}{3}\right] \quad [\text{Periodic Function}]$$
$$= \tan\frac{\pi}{3}$$
$$= \sqrt{3}$$

32. Prove $\text{Cos}\,4x = 1 - 8\text{Sin}^2 x.\text{Cos}^2 x$

Ans: Starting with the left-hand side and using the trigonometric addition formula,

$$\cos 2x = 1 - 2\sin^2 x$$

We get,

$$\text{L.H.S} = \text{Cos}\,4x$$

$=1-2\text{Sin}^2 2x$

$=1-2(\text{Sin}\,2x)^2$

$=1-2(2\text{Sin}\,x.\text{Cos}\,x)^2$

$=1-2(4\text{Sin}^2 x.\text{Cos}^2 x)$

$=1-8\text{Sin}^2 x.\text{Cos}^2 x$

33. Prove $\dfrac{\textbf{Cos}(\pi+\textbf{x}).\textbf{Cos}(-\textbf{x})}{\textbf{Sin}(\pi-\textbf{x}).\textbf{Cos}\left(\dfrac{\pi}{2}+\textbf{x}\right)}=\textbf{Cot}^2\textbf{x}$

Ans: Starting with the left-hand side and using the trigonometric periodic identities, we obtain the following

$$\text{L.H.S.}=\frac{\cos(\pi+x)\cos(-x)}{\sin(\pi-x)\cos\left(\frac{\pi}{2}+x\right)}$$

$$=\frac{-\cos x\cos x}{-\sin x\sin x}$$

$$=\cot^2 x$$

$$=\text{R.H.S.}$$

34. Prove that $\mathbf{\tan 56^\circ=\dfrac{Cos11^\circ+Sin11^\circ}{Cos11^\circ-Sin11^\circ}}$

Ans: Starting with the left-hand side and using the trigonometric addition formula for the tangent function, we obtain

$$\text{L.H.S.}=\tan 56^\circ$$

$$=\tan(45^\circ+11^\circ)$$

$$=\frac{\tan 45^\circ+\tan 11^\circ}{1-\tan 45^\circ\cdot\tan 11^\circ}$$

$$=\frac{1+\tan 11^\circ}{1-\tan 11^\circ}$$

$$=\frac{\cos 11^\circ+\sin 11^\circ}{\cos 11^\circ-\sin 11^\circ}$$

$= \text{R.H.S.}$

35. Prove that $\mathbf{Cos105^\circ + Cos15^\circ = Sin75^\circ - Sin15^\circ}$

Ans: Starting with the left-hand side and using the trigonometric difference formula for the cosine function, we obtain

$\text{L.H.S.} = \text{Cos}105^\circ + \text{Cos}15^\circ$

$= \text{Cos}(90^\circ + 15^\circ) + \text{Cos}(90^\circ - 75^\circ)$

$= -\text{Sin}15^\circ + \text{Sin}75^\circ$

$= \text{Sin}75^\circ - \text{Sin}15^\circ$

$= \text{R.H.S.}$

36. Find the value of $\mathbf{Cos(-1710^\circ)}$

Ans: We have $\cos\left(-1710^\circ\right)$. We also know $\cos\left(-x\right) = \cos x$

$\text{Cos}(-1710^\circ) = \text{Cos}(1800 - 90)$

$= \text{Cos}\left[5 \times 360 + 90\right]$

$= \text{Cos}\dfrac{\pi}{2}$

$= 0$

37. A wheel makes $\mathbf{360}$ revolutions in $\mathbf{1}$ minute. Through how many radians does it turn in $\mathbf{1}$ second.

Ans: Given,

Number of revolutions made in $60\text{s} = 360$

Number of revolutions made in $1\text{s} = \dfrac{360}{60}$

Angle moved in 6 revolutions $= 2\pi \times 6$

$= 12\pi$

38. Prove that $\mathbf{Sin^2 6x - Sin^2 4x = Sin2x.Sin10x}$

Ans: Starting with the left-hand side and using the trigonometric addition formula for the sine function, we obtain

$$\text{L.H.S.} = \text{Sin}^2 6x - \text{Sin}^2 4x$$
$$= \sin(6x+4x)\sin(6x-4x)$$
$$= \sin 10x \sin 2x$$
$$= \text{R.H.S.}$$

39. Prove that $\dfrac{\tan 69 + \tan 66}{1 \tan m 69 . \tan 66} = -1$

Ans: Starting with the left-hand side and using the trigonometric difference identity for the tangent function, we obtain

$$\text{L.H.S.} = \frac{\tan 69^\circ + \tan 66^\circ}{1 - \tan 69^\circ \tan 66^\circ}$$
$$= \tan(69^\circ + 66^\circ)$$
$$= \tan(135^\circ)$$
$$= \tan(90^\circ + 45^\circ)$$
$$= -1$$
$$= \text{R.H.S.}$$

40. Prove that $\dfrac{\text{Sin}\, x}{1 + \text{Cos}\, x} = \tan \dfrac{x}{2}$

Ans: Starting with the left-hand side and using the trigonometric addition identities for the sine and cosine function, we obtain

$$\text{L.H.S.} = \frac{\sin x}{1 + \cos x}$$
$$= \frac{2 \sin \frac{x}{2} \cos \frac{x}{2}}{2 \cos^2 \frac{x}{2}}$$
$$= \frac{\sin \frac{x}{2}}{\cos \frac{x}{2}}$$

$= \tan \frac{x}{2}$

$= \text{R.H.S.}$

Long Answer Questions **4 Marks**

Prove the following identities

1. The minute hand of a watch is 1.5cm long. How far does its tip move in 40 minute?

Ans: Analysing the given information, we have

$r = 1.5\text{cm}$

$\text{Angle made in } 60\text{min} = 360^\circ$

$\text{Angle made in } 1\text{min} = 6^\circ$

$\text{Angle made in } 40\text{min} = 6^\circ \times 40^\circ = 240^\circ$

Calculating the arc distance

$\theta = \frac{l}{r}$

$240 \times \frac{\pi}{180} = \frac{l}{1.5}$

$2 \times 3.14 = l$

$6.28 = l$

$l = 6.28\text{cm}$

2. Show that tan 3x. tan 2x. tan x = tan 3x – tan 2x – tan x

Ans: Let us start with $\tan 3x$ and we know $3x = 2x + x$

$\tan 3x = \tan(2x + x)$

$\frac{\tan 3x}{1} = \frac{\tan 2x + \tan x}{1 - \tan 2x.\tan x}$

$\tan 3x(1 - \tan 2x.\tan x) = \tan 2x + \tan x$

$\tan 3x - \tan 3x.\tan 2x.\tan x = \tan 2x + \tan x$

$\tan 3x.\tan 2x.\tan x = \tan 3x - \tan 2x - \tan x$

3. Find the value of $\tan\frac{\pi}{8}$

Ans: We know that

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

Therefore, we have

$$\tan\left(2\frac{\pi}{8}\right) = \frac{2\tan\frac{\pi}{8}}{1-\tan^2\frac{\pi}{8}}$$

$$\Rightarrow 1 = \frac{2\tan\frac{\pi}{8}}{1-\tan^2\frac{\pi}{8}}$$

Put $\tan\frac{\pi}{8} = x$

$$1 = \frac{2x}{1-x^2}$$

$$\Rightarrow 2x = 1-x^2$$

$$\Rightarrow x = \frac{-1\pm\sqrt{2}}{1}$$

Since, $\frac{\pi}{8}$ lies in the first quadrant, the value must be positive, hence

$$\tan\frac{\pi}{8} = \sqrt{2}-1$$

4. Prove that $\frac{\mathrm{Sin}(x+y)}{\mathrm{Sin}(x-y)} = \frac{\tan x+\tan y}{\tan x-\tan y}$

Ans: Starting with the left-hand side and using the trigonometric difference formula for the sine function, we get

$$\text{L.H.S.} = \frac{\mathrm{Sin}(x+y)}{\mathrm{Sin}(x-y)}$$

$$=\frac{\operatorname{Sin} x.\operatorname{Cos} y+\operatorname{Cos} x.\operatorname{Sin} y}{\operatorname{Sin} x.\operatorname{Cos} y-\operatorname{Cos} x.\operatorname{Sin} y}$$

Dividing numerator and denominator by $\operatorname{Cos} x.\operatorname{Cos} y$

$$=\frac{\tan x+\tan y}{\tan x-\tan y}$$

$=\text{R.H.S.}$

5. If in two circles, arcs of the same length subtend angles 60° and 75° at the center find the ratio of their radii.

Ans: We know that the length of the arc and its subtended angle is related using the following formula

$$\theta=\frac{1}{r_1}$$

Therefore, we have

$$60\times\frac{\pi}{18}=\frac{1}{r_1}$$

$$r_1=\frac{3l}{\pi} \qquad \text{.....}(1)$$

$$\theta=\frac{1}{r_2}$$

$$75\times\frac{\pi}{18}=\frac{1}{r_2}$$

$$r_2=\frac{12l}{5\pi} \qquad \text{.....}(2)$$

$(1)\div(2)$

$$\frac{r_1}{r_2}=\frac{\frac{3l}{\pi}}{\frac{12l}{5\pi}}$$

$$=\frac{31}{\pi}\times\frac{5\pi}{12l}$$

$=5:4$

6. Prove that $\mathbf{Cos\,6x = 32Cos^2 x - 48Cos^4 x + 18Cos^2 x - 1}$

Ans: Starting with the left-hand side and using the trigonometric identities for the cosine function, we obtain

$\text{L.H.S.} = \text{Cos}\,6x$

$= \text{Cos}\,2(3x) = 2\text{Cos}^2 3x - 1$

$= \text{Cos}\,2(3x)$

$= 2(4\cos^3 x - 3\cos x)^2 - 1$

$= 2\left[16\text{Cos}^6 x + 9\text{Cos}^2 x - 24\text{Cos}^4 x\right] - 1$

$= 32\text{Cos}^6 x + 18\text{Cos}^2 x - 48\text{Cos}^4 x - 1$

$= 32\text{Cos}^6 x - 48\text{Cos}^4 x + 18\text{Cos}^2 x1$

$= \text{R.H.S.}$

7. Solve $\mathbf{Sin\,2x - Sin\,4x + Sin\,6x = 0}$

Ans: Starting with the left-hand side and using the trigonometric addition identity for the sine function, we obtain

$\text{L.H.S.} = \text{Sin}\,6x + \text{Sin}\,2x - \text{Sin}\,4x$

$= 2\sin\left(\dfrac{6x + 2x}{2}\right)\cos\left(\dfrac{6x - 2x}{2}\right) - \sin 4x$

$= \sin 4x\left(2\cos 2x - 1\right)$

$= 0$

Now,

$\sin 4x = 0$

$4x = n\pi$

$x = \dfrac{n\pi}{4}$

Also,

$2\cos 2x - 1 = 0$

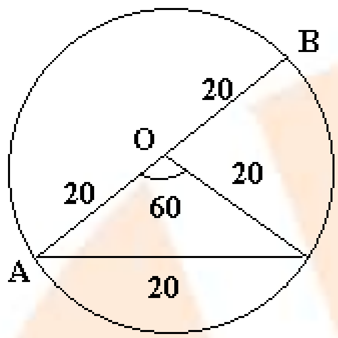
$\cos 2x = \cos\dfrac{\pi}{3}$

$2x = 2n\pi \pm \dfrac{\pi}{3}$

$x = n\pi \pm \dfrac{\pi}{6}$

8. In a circle of diameter 40cm, the length of a chord is 20cm. Find the length of minor are of the chord.

Ans: Given,



$$\theta = \frac{l}{r}$$

$$\Rightarrow 60 \times \frac{\pi}{180} = \frac{l}{20}$$

$$\Rightarrow l = \frac{20\pi}{3}\text{cm/s}$$

9. Prove that $\tan 4x = \dfrac{4\tan x(1-\tan^2 x)}{1-6\tan^2 x+\tan^4 x}$

Ans: Starting with the left-hand side and using the trigonometric addition identities for the tangent function, we obtain

$$\text{L.H.S.} = \tan 4x$$

$$= \frac{2\tan 2x}{1-\tan^2 2x}$$

$$= \frac{2.\dfrac{2\tan 2x}{1-\tan^2 2x}}{1-\left(\dfrac{2\tan 2x}{1-\tan^2 2x}\right)^2}$$

$$= \frac{\dfrac{4\tan x}{1-\tan^2 x}}{\dfrac{(1-\tan^2 x)^2-4\tan^2 x}{(1-\tan^2 x)^2}}$$

$$= \frac{4\tan x}{(1-\tan^2 x)} \times \frac{(1-\tan^2 x)}{1+\tan^4 x - 2\tan^2 x - 4\tan^2 x}$$
$$= \frac{4\tan x(1-\tan^2 x)}{1-6\tan^2 x+\tan^4 x}$$
$$= \text{R.H.S.}$$

10. Prove that $\left(\mathbf{Cosx+Cosy}\right)^2+\left(\mathbf{Sinx-Siny}\right)^2=\mathbf{4Cos^2}\left(\frac{\mathbf{x+y}}{\mathbf{2}}\right)$

Ans: Starting with the left-hand side and using the trigonometric addition identities for the cosine and sine function, we obtain

$$\text{L.H.S.} = \left(\text{Cosx}+\text{Cosy}\right)^2+\left(\text{Sinx}-\text{Siny}\right)^2$$
$$= \left(2\text{Cos}\frac{x+y}{2}.\text{Cos}\frac{x-y}{2}\right)^2 + \left(2\text{Cos}\left(\frac{x+y}{2}\right).\text{Sin}\left(\frac{x-y}{2}\right)\right)^2$$
$$= 4\text{Cos}^2\frac{x+y}{2}.\text{Cos}^2\left(\frac{x-y}{2}\right)+4\text{Cos}^2\frac{x+y}{2}.\text{Sin}^2\frac{x-y}{2}$$
$$= 4\text{Cos}^2\left(\frac{x+y}{2}\right)\left[\text{Cos}^2\frac{x-y}{2}+\text{Sin}^2\frac{x-y}{2}\right]$$
$$= 4\text{Cos}^2\left(\frac{x+y}{2}\right)$$
$$= \text{R.H.S.}$$

11. If $\mathbf{Cotx=-\frac{5}{12}}$, x lies in second quadrant find the values of other five trigonometric functions

Ans: Given

$$\text{Cotx} = -\frac{5}{12}$$

Using some trigonometric identities, we obtain

$$\tan x = -\frac{12}{5}$$
$$\text{Sec}^2 x = 1+\tan^2 x$$
$$\text{Sec}\, x = \pm\frac{13}{5}$$

Since x lies in the second quadrant, the cosine value will be negative

$$\operatorname{Sec} x = -\frac{13}{5}$$

$$\operatorname{Cos} x = -\frac{5}{13}$$

$$\operatorname{Sin} x = \tan x . \operatorname{Cos} x$$

$$= \frac{-12}{5} \times \left(\frac{-5}{13}\right)$$

$$= \frac{12}{13}$$

$$\operatorname{Csc} x = \frac{13}{12}$$

12. Prove that $\frac{\operatorname{Sin} 5x - 2\operatorname{Sin} 3x + \operatorname{Sin} x}{\operatorname{Cos} 5x - \operatorname{Cos} x} = \tan x$

Ans: Starting with the left-hand side and using the trigonometric difference identities for the sine function, we obtain

$$\text{L.H.S.} = \frac{\operatorname{Sin} 5x + \operatorname{Sin} x - 2\operatorname{Sin} 3x}{\operatorname{Cos} 5x - \operatorname{Cos} x}$$

$$= \frac{2\operatorname{Sin} 3x . \operatorname{Cos} 2x - 2\operatorname{Sin} 3x}{-2\operatorname{Sin} 3x . \operatorname{Sin} 2x}$$

$$= \frac{2\operatorname{Sin} 3x(\operatorname{Cos} 2x - 1)}{-2\operatorname{Sin} 3x . \operatorname{Sin} 2x}$$

$$= \frac{-(1 - \operatorname{Cos} 2x)}{-\operatorname{Sin} 2x}$$

$$= \frac{2\operatorname{Sin}^2 x}{2\operatorname{Sin} x . \operatorname{Cos} x}$$

$$= \frac{\operatorname{Sin} x}{\operatorname{Cos} x}$$

$$= \tan x$$

$$= \text{R.H.S.}$$

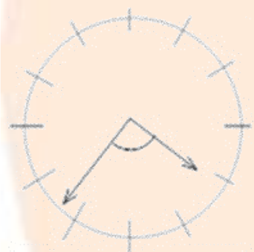
13. Prove that $\operatorname{Sin}x + \operatorname{Sin}3x + \operatorname{Sin}5x + \operatorname{Sin}7x = 4\operatorname{Cos}x.\operatorname{Cos}2x.\operatorname{Sin}4x$

Ans: Starting with the left-hand side and using the trigonometric addition identities for the sine function, we obtain

$$\text{L.H.S.} = \text{Sinx} + \text{Sin3x} + \text{Sin5x} + \text{Sin7x}$$
$$= \text{Sin}\,x + \text{Sin}\,7x + \text{Sin}\,3x + \text{Sin}\,5x$$
$$= 2\text{Sin}\left(\frac{x+7x}{2}\right).\text{Cos}\left(\frac{x-7x}{2}\right) + 2\text{Sin}\left(\frac{3x+5x}{2}\right)\text{Cos}\left(\frac{3x-5x}{2}\right)$$
$$= 2\text{Sin}\,4x.\text{Cos}\,3x + 2\text{Sin}\,4x.\text{Cos}\,x$$
$$= 2\text{Sin}\,4x[\text{Cos}\,3x + \text{Cos}\,x]$$
$$= 2\text{Sin}\,4x\left[2\text{Cos}\left(\frac{3x+x}{2}\right).\text{Cos}\left(\frac{3x-x}{2}\right)\right]$$
$$= 2\text{Sin}\,4x[2\text{Cos}\,2x.\text{Cos}\,x]$$
$$= 4\text{Cos}\,x.\text{Cos}\,2x.\text{Sin}\,4x$$
$$= \text{R.H.S.}$$

14. Find the angle between the minute hand and hour hand of a clock when the time is 7.20

Ans: We know that the angle made by minute hand in $15\text{min} = 15 \times 6 = 90^\circ$



We also know that the angle made by the hour hand in $1\text{hr} = 30^\circ$

In 60 minute $= \frac{30}{60}$

$= \frac{1}{2}$

[$\because$ Angle Travelled by hr hand in $12\text{hr} = 360^\circ$]

In 20 minutes $= \frac{1}{2} \times 20$

$= 10^\circ$

Angle made $= 90 + 10$

$= 100^\circ$

15. Show that $\sqrt{2+\sqrt{2+2\text{Cos}\,4\theta}} = 2\text{Cos}\,\theta$

Ans: Starting with the left-hand side and using the trigonometric addition identity for the cosine function, we obtain

$$\text{L.H.S.} = \sqrt{2+\sqrt{2+2\text{Cos}\,4\theta}}$$
$$= \sqrt{2+\sqrt{2(1+\text{Cos}\,4\theta)}}$$
$$= \sqrt{2+\sqrt{2.2\text{Cos}^2\,2\theta}}$$
$$= \sqrt{2+2\text{Cos}\,2\theta}$$
$$= \sqrt{2(1+\text{Cos}\,2\theta)}$$
$$= \sqrt{2.2\text{Cos}^2\,\theta}$$
$$= 2\text{Cos}\,\theta$$
$$= \text{R.H.S.}$$

16. Prove that $\mathbf{Cot4x\left(Sin5x+Sin3x\right)=Cotx\left(Sin5x-Sin3x\right)}$

Ans: Starting with the left-hand side and using the trigonometric addition identity for the sine function, we obtain

$$\text{L.H.S.} = \text{Cot}4\text{x}\left(\text{Sin}5\text{x}+\text{Sin}3\text{x}\right)$$
$$= \frac{\text{Cos}\,4\text{x}}{\text{Sin}\,4\text{x}}\left[2\text{Sin}\frac{5\text{x}+3\text{x}}{2}.\text{Cos}\frac{5\text{x}-3\text{x}}{2}\right]$$
$$= \frac{\text{Cos}\,4\text{x}}{\text{Sin}\,4\text{x}}2\text{Sin}\,4\text{x}.\text{Cos}\,\text{x}$$
$$= 2\text{Cos}\,4\text{x}.\text{Cos}\,\text{x}$$

Then, we move on to the right-hand side and using the trigonometric addition identity for the sine function, we obtain

$$\text{R.H.S.} = \text{Cotx}\left(\text{Sin}5\text{x}-\text{Sin}3\text{x}\right)$$
$$= \frac{\text{Cos}\,\text{x}}{\text{Sin}\,\text{x}}\left[2\text{Cos}\frac{5\text{x}+3\text{x}}{2}.\text{Sin}\frac{5\text{x}-3\text{x}}{2}\right]$$
$$= \frac{\text{Cos}\,\text{x}}{\text{Sin}\,\text{x}}[2\text{Cos}\,4\text{x}.\text{Sin}\,\text{x}]$$
$$= 2\text{Cos}\,4\text{x}.\text{Cos}\,\text{x}$$

Therefore,

$$\text{L.H.S} = \text{R.H.S}$$

Long Answer Questions **6 Marks**

1. Find the general solution of $\sin 2x + \sin 4x + \sin 6x = 0$

Ans: We have that $\sin 2x + \sin 4x + \sin 6x = 0$

$\Rightarrow (\sin 2x + \sin 6x) + \sin 4x = 0$

$\Rightarrow \left(2\sin\left(\frac{2x+6x}{2}\right)\cos\left(\frac{2x-6x}{2}\right)\right) + \sin 4x = 0$

$\Rightarrow 2\sin 4x \cos 2x + \sin 4x = 0$

$\Rightarrow \sin 4x(2\cos 2x + 1) = 0$

Now

$\sin 4x = 0$

$\Rightarrow x = n\pi$

$2\cos 2x + 1 = 0$

$\Rightarrow x = n\pi \pm \frac{\pi}{3}$

2. Find the general solution of $\cos\theta\cos 2\theta\cos 3\theta = \frac{1}{4}$

Ans: We have that $\cos\theta\cos 2\theta\cos 3\theta = \frac{1}{4}$

$\Rightarrow 4\cos\theta\cos 2\theta\cos 3\theta = 1$

Using the trigonometric addition identity for the cosine function, we obtain

$\Rightarrow 2(2\cos\theta\cos 3\theta)\cos 2\theta - 1 = 0$

$\Rightarrow 2(\cos 4\theta + \cos 2\theta)\cos 2\theta - 1 = 0$

$\Rightarrow 2(2\cos^2 2\theta - 1 + \cos 2\theta)\cos 2\theta - 1 = 0$

$\Rightarrow (2\cos^2 2\theta - 1)(2\cos 2\theta + 1) = 0$

Now,

$2\cos^2 2\theta - 1 = 0$

$\Rightarrow \cos 4\theta = 0$

$\Rightarrow 4\theta = (2n+1)\frac{\pi}{2}$

$\Rightarrow \theta = (2n+1)\frac{\pi}{8}$

Also,

$2\cos 2\theta + 1 = 0$

$$\Rightarrow \cos 2\theta = -\frac{1}{2}$$

$$\Rightarrow \theta = n\pi \pm \frac{\pi}{3}$$

3. If $\mathbf{Sin\alpha + Sin\beta = a}$ and $\mathbf{Cos\alpha + Cos\beta = b}$

Show that $\mathbf{Cos(\alpha+\beta) = \frac{b^2 - a^2}{b^2 + a^2}}$

Ans: Squaring both the equations and adding them together,

$$b^2 + a^2 = (Cos\alpha + Cos\beta)^2 + (Sin\alpha + Sin\beta)^2$$

$$= Cos^2\alpha + Cos^2\beta + 2Cos\alpha.Cos\beta + Sin^2\alpha + Sin^2\beta + 2Sin\alpha.Sin\beta$$

$$= 1 + 1 + 2(Cos\alpha.Cos\beta + Sin\alpha.Sin\beta)$$

$$= 2 + 2Cos(\alpha - \beta) \qquad (1)$$

$$b^2 - a^2 = (Cos\alpha + Cos\beta)^2 - (Sin\alpha + Sin\beta)^2$$

$$= (Cos^2\alpha - Sin^2\beta) + (Cos^2\beta - Sin^2\alpha) + 2Cos(\alpha+\beta)$$

$$= Cos(\alpha+\beta)Cos(\alpha-\beta) + Cos(\beta+\alpha)Cos(\alpha-\beta) + 2Cos(\alpha+\beta)$$

$$= 2Cos(\alpha+\beta).Cos(\alpha-\beta) + 2Cos(\alpha+\beta)$$

$$= Cos(\alpha+\beta)[2Cos(\alpha-\beta) + 2]$$

$$= Cos(\alpha+\beta).(b^2 + a^2) \qquad \text{from (1)}$$

Dividing equation (1) with $b^2 + a^2$, we get

$$\frac{b^2 - a^2}{b^2 + a^2} = Cos(\alpha+\beta)$$

4. Prove

$$\mathbf{Cos\alpha + Cos\beta + Cos\gamma + Cos(\alpha+\beta+\gamma) = 4Cos\left(\frac{\alpha+\beta}{2}\right).Cos\left(\frac{\beta+\gamma}{2}\right).Cos\left(\frac{\gamma+\alpha}{2}\right)}$$

Ans: Starting with the left-hand side and using the trigonometric addition identities for the cosine function, we obtain

$$L.H.S. = Cos\alpha + Cos\beta + Cos\gamma + Cos(\alpha+\beta+\gamma)$$

$$= 2Cos\left(\frac{\alpha+\beta}{2}\right).Cos\left(\frac{\alpha-\beta}{2}\right) + 2Cos\left(\frac{\alpha+\beta+\gamma+\gamma}{2}\right).Cos\left(\frac{\alpha+\beta+\gamma-\gamma}{2}\right)$$

$$=2\operatorname{Cos}\left(\frac{\alpha+\beta}{2}\right).\operatorname{Cos}\left(\frac{\alpha-\beta}{2}\right)+2\operatorname{Cos}\left(\frac{\alpha+\beta}{2}\right).\operatorname{Cos}\left(\frac{\alpha+\beta+2\gamma}{2}\right)$$

$$=2\operatorname{Cos}\left(\frac{\alpha+\beta}{2}\right)\left[\operatorname{Cos}\left(\frac{\alpha-\beta}{2}\right)+\operatorname{Cos}\left(\frac{\alpha+\beta+2\gamma}{2}\right)\right]$$

$$=2\operatorname{Cos}\left(\frac{\alpha+\beta}{2}\right)\left[2\operatorname{Cos}\left(\frac{\frac{\alpha-\beta}{2}+\frac{\alpha+\beta+2\gamma}{2}}{2}\right).\operatorname{Cos}\left(\frac{\frac{\alpha+\beta+2\gamma}{2}-\frac{\alpha-\beta}{2}}{2}\right)\right]$$

$$=2\operatorname{Cos}\left(\frac{\alpha+\beta}{2}\right)\left[2\operatorname{Cos}\left(\frac{\alpha+\gamma}{2}\right).\operatorname{Cos}\left(\frac{\beta+\gamma}{2}\right)\right]$$

$$=4\operatorname{Cos}\left(\frac{\alpha+\beta}{2}\right).\operatorname{Cos}\left(\frac{\beta+\gamma}{2}\right).\operatorname{Cos}\left(\frac{\gamma+\alpha}{2}\right)$$

$=$ R.H.S.

5. Prove that $\mathbf{Sin\,3x+Sin\,2x-Sin\,2x=4Sin\,x.Cos\frac{x}{2}.Cos\frac{3x}{2}}$

Ans: Starting with the left-hand side and using the trigonometric addition identity for the sine function, we obtain

$\text{L.H.S.}=\sin 3x+\sin x-\sin 2x$

$$=2\cos\left(\frac{3x+x}{2}\right).\operatorname{Sin}\left(\frac{3x+x}{2}\right)+\operatorname{Sin}2x$$

$=2\cos 2x.\sin x+\sin 2x$

$=2\cos 2x.\sin x+2\sin x\cos x$

$=2\sin x[\cos 2x+\cos x]$

$$=2\sin x\left[2\cos x\frac{3x}{2}.\cos\frac{x}{2}\right]$$

$$=4\sin x\cos x\frac{3x}{2}\cos\frac{x}{2}$$

$=$ R.H.S.

6. Prove that $\mathbf{2\cos\frac{\pi}{13}.\cos\frac{9\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}=0}$

Ans: Starting with the left-hand side using the trigonometric addition identities for the cosine and sine function, we obtain

$$\begin{aligned}
\text{L.H.S.} &= 2\cos\frac{\pi}{13}.\cos\frac{9\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}\\
&= \cos\left(\frac{\pi}{13}+\frac{9\pi}{13}\right)+\cos\left(\frac{\pi}{13}-\frac{9\pi}{13}\right)+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}\\
&= \cos\frac{10\pi}{13}+\cos\frac{18\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}\\
&= \cos\left(\pi-\frac{3\pi}{13}\right)+\cos\left(\pi-\frac{5\pi}{13}\right)+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}\\
&= -\cos\frac{3\pi}{13}-\cos\frac{5\pi}{13}+\frac{3\pi}{13}+\cos\frac{5\pi}{13}\\
&= 0\\
&= \text{R.H.S.}
\end{aligned}$$

7. Find the value of $\tan(\alpha+\beta)$ given that $\cot\alpha=\frac{1}{2},\alpha\in\left(\pi,\frac{3\pi}{2}\right)$ and $\text{Sec}\beta=-\frac{5}{3},\beta\in\left(\frac{\pi}{2},\pi\right)$

Ans: We know that,

$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

Given,

$$\text{Cot}\alpha=\frac{1}{2}$$

$$\tan\alpha=2$$

Now, let us find $\tan\beta$

$$1+\tan^2\beta=\text{Sec}^2\beta$$

$$1+\tan^2\beta=\left(\frac{-5}{3}\right)^2\left[\because \text{Sec}\beta=\frac{-5}{3}\right]$$

$$\tan\beta=\pm\frac{4}{3}$$

$$\tan\beta = -\frac{4}{3}\left[\because \beta \in \left(\frac{\frac{\pi}{2}}{x}\right)\right]$$

Therefore, we have that

$$\tan(\alpha+\beta) = \frac{2-\frac{4}{3}}{1-2\left(\frac{-4}{3}\right)}$$

$$= \frac{2}{11}$$

8. Prove that $\frac{\mathbf{Sec8A-1}}{\mathbf{Sec4A-1}} = \frac{\mathbf{tan8A}}{\mathbf{tan2A}}$

Ans: Starting with the left-hand side and using the trigonometric elementary identities of the cosine function and sine function, we obtain

$$\text{L.H.S.} = \frac{\sec 8A - 1}{\sec 4A - 1}$$

$$= \frac{\frac{1}{\text{Cos}8A} - 1}{\frac{1}{\text{Cos}4A} - 1}$$

$$= \frac{1-\text{Cos}8A}{1-\text{Cos}4A} \times \frac{\text{Cos}4A}{\text{Cos}8A}$$

$$= \frac{2\text{Sin}^2 4A}{2\text{Sin}^2 2A} \cdot \frac{\text{Cos}4A}{\text{Cos}8A}$$

$$= \frac{(2\text{Sin}4A.\text{Cos}4A).\text{Sin}4A}{2\text{Sin}^2 2A.\text{Cos}8A}$$

$$= \frac{\text{Sin}8A(2\text{Sin}2A.\text{Cos}2A)}{2\text{Sin}^2 2A.\text{Cos}8A}$$

$$= \frac{\text{Sin}8A\,\text{Cos}2A}{\text{Sin}2A.\text{Cos}2A}$$

$$= \frac{\tan 8A}{\tan 2A}$$

$=\text{R.H.S.}$

9. Prove that $\mathbf{\cos^2 x+\cos^2\left(x+\frac{\pi}{3}\right)+\cos^2\left(x-\frac{\pi}{3}\right)=\frac{3}{2}}$

Ans: Starting with the left-hand side and using trigonometric addition identities of the cosine function, we obtain

$$\text{L.H.S.}=\frac{1+\cos 2x}{2}+\frac{1+\cos\left(2x+\frac{2\pi}{3}\right)}{2}+\frac{1+\cos\left(2x-\frac{2\pi}{3}\right)}{2}$$

$$=\frac{1}{2}\left[1+1+1+\cos 2x+\cos\left(2x+\frac{2\pi}{3}\right)+\cos\left(2x-\frac{2\pi}{3}\right)\right]$$

$$=\frac{1}{2}\left[3+\cos 2x+\cos\left(2x+\frac{2\pi}{3}\right)+\cos\left(2x-\frac{2\pi}{3}\right)\right]$$

$$=\frac{1}{2}\left[3+\cos 2x+2\cos\left(\frac{2x+\frac{2\pi}{3}+2x-\frac{2\pi}{3}}{2}\right).\cos\left(\frac{2x+\frac{2\pi}{3}-2x+\frac{2\pi}{3}}{2}\right)\right]$$

$$=\frac{1}{2}\left[3+\cos 2x+2\cos 2x.\cos\frac{4\pi}{6}\right]$$

$$=\frac{1}{2}\left[3+\cos 2x+2\cos 2x.\cos\frac{2\pi}{3}\right]$$

$$=\frac{1}{2}\left[3+\cos 2x+2\cos 2x.\cos\left(\pi-\frac{\pi}{3}\right)\right]$$

$$=\frac{1}{2}\left[3+\cos 2x+2\cos 2x.\left(\frac{-1}{2}\right)\right]$$

$$=\frac{3}{2}$$

$=\text{R.H.S.}$

10. Prove that $\mathbf{\cos 2x.\cos\frac{x}{2}-\cos 3x.\cos\frac{9x}{2}=\sin 5x\sin\frac{5x}{2}}$

Ans: Starting with the left-hand side and using trigonometric addition identities for the cosine function, we obtain

$$\text{L.H.S.} = \frac{1}{2}\left[2\text{Cos}\,2x.\text{Cos}\frac{x}{2} - 2\text{Cos}\,3x.\text{Cos}\frac{9x}{2}\right]$$

$$= \frac{1}{2}\left[\text{Cos}\left(2x + \frac{x}{2}\right) + \text{Cos}\left(2x - \frac{x}{2}\right) - \text{Cos}\left(\frac{9x}{2} + 3x\right) - \text{Cos}\left(\frac{9x}{2} - 3x\right)\right]$$

$$= \frac{1}{2}\left[\text{Cos}\frac{5x}{2} + \text{Cos}\frac{3x}{2} - \text{Cos}\frac{15x}{2} - \text{Cos}\frac{3x}{2}\right]$$

$$= \frac{1}{2}\left[\text{Cos}\frac{5x}{2} - \text{Cos}\frac{15x}{2}\right]$$

$$= \frac{1}{2}\left[-2\text{Sin}\left(\frac{\frac{5x}{2} + \frac{15x}{2}}{2}\right).\text{Sin}\left(\frac{\frac{5x}{2} - \frac{15x}{2}}{2}\right)\right]$$

$$= -\text{Sin}\,5x.\text{Sin}\left(\frac{-5x}{2}\right)$$

$$= \text{Sin}\,5x.\text{Sin}\frac{5x}{2}$$

$$= \text{R.H.S.}$$

11. Prove that $\mathbf{Cos\,20°.Cos\,40°.Cos\,60°.Cos\,80° = \frac{1}{16}}$

Ans: Starting with the left-hand side and using the trigonometric addition identities of the cosine function, we obtain

$$\text{L.H.S.} = \text{Cos}\,20^\circ.\text{Cos}\,40^\circ.\text{Cos}\,60^\circ.\text{Cos}\,80^\circ$$

$$= \text{Cos}\,60^\circ.\text{Cos}\,20^\circ.\text{Cos}\,40^\circ.\text{Cos}\,80^\circ$$

$$= \frac{1}{2}.\frac{1}{2}\text{Cos}\,40^\circ\left(2\text{Cos}\,20^\circ.\text{Cos}\,80^\circ\right)$$

$$= \frac{1}{4}\text{Cos}\,40^\circ\left[\text{Cos}(80+20) + \text{Cos}(80-20)\right]$$

$$= \frac{1}{4}\text{Cos}\,40^\circ\left[\text{Cos}100^\circ + \text{Cos}\,60^\circ\right]$$

$$=\frac{1}{4}\text{Cos}\,40^{\circ}\left[\text{Cos}100^{\circ}+\frac{1}{2}\right]$$

$$=\frac{1}{8}(2\text{Cos}100^{\circ}\,\text{Cos}\,40^{\circ})+\frac{1}{8}\text{Cos}\,40^{\circ}$$

$$=\frac{1}{8}\left[\text{Cos}(100+40)^{\circ}+\text{Cos}(100-40)^{\circ}\right]+\frac{1}{8}\text{Cos}\,40^{\circ}$$

$$=\frac{1}{8}\left[\text{Cos}140^{\circ}+\text{Cos}\,60^{\circ}\right]+\frac{1}{8}\text{Cos}\,40^{\circ}$$

$$=\frac{1}{8}\left[\text{Cos}140^{\circ}+\frac{1}{2}\right]+\frac{1}{8}\text{Cos}\,40^{\circ}$$

$$=\frac{1}{8}\text{Cos}(180-40)^{\circ}+\frac{1}{16}+\frac{1}{8}\text{Cos}\,40^{\circ}$$

$$=-\frac{1}{8}\text{Cos}\,40^{\circ}+\frac{1}{16}+\frac{1}{8}\text{Cos}\,40^{\circ}$$

$$=\frac{1}{16}$$

$$=\text{R.H.S.}$$

12. If $\tan x=\frac{3}{4},\pi<x<\frac{3\pi}{2}$, Find the value of $\text{Sin}\frac{x}{2},\text{Cos}\frac{x}{2}$ and $\tan\frac{x}{2}$

Ans: Given that

$\pi<x<\frac{3\pi}{2}$ implying that x is in the third quadrant

$$\Rightarrow\frac{\pi}{2}<\frac{x}{2}<\frac{3\pi}{2}$$

Therefore, we have that $\text{Sin}\frac{x}{2}$ is positive and $\text{Cos}\frac{x}{2}$ is negative.

Let us find for $\tan\frac{x}{2}$

We know

$$1+\tan^2 x=\text{Sec}^2 x\frac{5}{4}$$

$$1+\left(\frac{3}{4}\right)^2=\mathrm{Sec}^2 x$$

$$\mathrm{Sec}^2 x=\pm\frac{25}{16}$$

$$\mathrm{Cos}\, x=\pm\frac{4}{5}$$

$$\mathrm{Cos}\, x=-\frac{4}{5} \qquad \left[\because \pi<x<\frac{3\pi}{2}\right]$$

Let us find the required values

$$\mathrm{Sin}\frac{x}{2}=\sqrt{\frac{1-\mathrm{Cos}\, x}{2}}$$

$$=\sqrt{\frac{1+\frac{4}{5}}{2}}$$

$$=\sqrt{\frac{9}{10}}$$

$$=\frac{3}{\sqrt{10}}$$

$$\mathrm{Cos}\frac{x}{2}=-\sqrt{\frac{1-\mathrm{Cos}\, x}{2}}$$

$$=-\sqrt{\frac{1-\frac{4}{5}}{2}}$$

$$=-\sqrt{\frac{1}{10}}$$

$$=\frac{-1}{\sqrt{10}}$$

$$\tan\frac{x}{2}=\frac{\frac{3}{\sqrt{10}}}{\frac{-1}{\sqrt{10}}}$$

$$=-3$$