Revision Notes

Class 11 Physics

Chapter 14 – Oscillation and waves

1. INTRODUCTION

1) Periodic motion refers to the type of motion which repeats itself over and over again after regular intervals of time.
2) Oscillatory or vibratory motion refers to the type of motion in which an object moves to and fro or back and forth in a repetitive manner about a fixed point in a definite interval of time.
3) Simple harmonic motion can be considered as a specific type of oscillatory motion, in which
   a) the particle moves in a single dimension
   b) the particle oscillates to and fro about a fixed mean position (where $F_{net} = 0$),
   c) the net force on the particle always gets directed towards the equilibrium position
   d) the magnitude of net force is always proportional to the displacement of the particle from the equilibrium position at that instant.

   Mathematically,

   $F_{net} = -kx$

   where, k is known as force constant

   $\Rightarrow ma = -kx$

   $\Rightarrow a = -\frac{kx}{m}$

   However, $a = -\omega^2 x$

   where, ⌉ is known as angular frequency

   $\Rightarrow \frac{d^2x}{dt^2} = -\omega^2 x$

   This equation is known as the differential equation of S.H.M.

   The general expression for $x(t)$ satisfying the above equation is:

   $x(t) = A\sin(\omega t + \phi)$

1.1 Some Important terms

1. Amplitude

The amplitude of a particle executing S.H.M. refers to its maximum displacement on either side of the equilibrium position. Amplitude of a particle is represented by A.

2. **Time Period**

Time period of a particle executing S.H.M. refers to the time taken to complete one cycle. It is represented by T. Mathematically,

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} \left(\because \omega = \sqrt{\frac{m}{k}} \right)$$

3. **Frequency**

The frequency of a particle executing S.H.M. is the same as the number of oscillations completed in one second. It is denoted by ν. Mathematically,

$$\nu = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

4. **Phase**

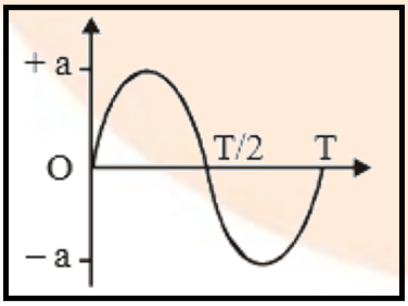
The phase of particle executing S.H.M. at any instant refers to its state with respect to its position and direction of motion at that particular instant. It is measured as argument (angle) of sine in the equation of S.H.M.

$\text{Phase} = (\omega t + \phi)$

When $t = 0$, $\text{phase} = \phi$; the constant ϕ is called initial phase of the particle or phase constant.

1.2 Important Relations

1. **Position**

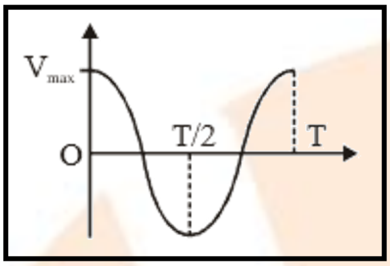


When the equilibrium position is considered at origin, the position (x coordinate) depends on time in general as $x(t) = \sin(\omega t + \phi)$.

- At the equilibrium position, $x = 0$

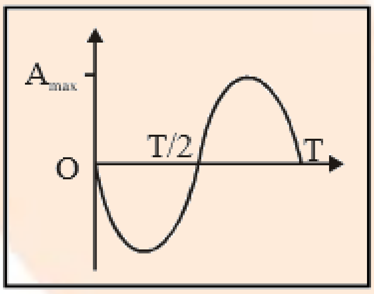
- At the extremes, $x = +a, -a$.

2. Velocity



- At any instant t, $v(t) = A\omega\cos(\omega t + \phi)$
- At any position x, $v(x) = \pm\omega\sqrt{A^2 - x^2}$
- Velocity has minimum magnitude at the extremes since the particle is at rest here.
  i.e., $v = 0$ at extreme position.
- On the other hand, velocity has maximum magnitude at the equilibrium position.
  i.e., $|v|_{max} = \omega A$ at equilibrium position.

3. Acceleration

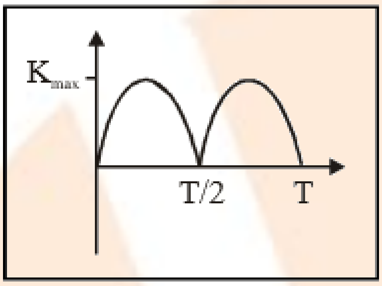


- At any instant t, $a(t) = -\omega^2 A \sin(\omega t + \phi)$
- At any position x, $a(x) = -\omega^2 x$
- Acceleration is always directed towards the equilibrium position.
- The magnitude of acceleration is minimum at equilibrium position and maximum at extremes.
  $|a|_{min} = 0$ at equilibrium position
  $|a|_{max} = \omega^2 A$ at extremes

4. Energy

Kinetic energy

- $K = \frac{1}{2}mv^2 \Rightarrow K = \frac{1}{2}m\omega^2\left(A^2 - x^2\right) = \frac{1}{2}m\omega^2A^2\cos^2\left(\omega t + \phi\right)$

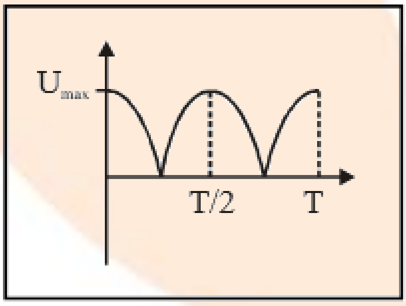


- K is maximum at equilibrium position and minimum at extremes.
- $K_{max} = \frac{1}{2}m\omega^2A^2 = \frac{1}{2}kA^2$ at equilibrium position.
- $K_{min} = 0$ at extremes.

Potential Energy

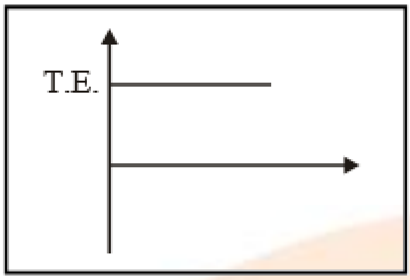
- If potential energy is taken as zero at equilibrium position, then at any position x,

  $U\left(x\right) = \frac{1}{2}kx^2 = \frac{1}{2}mA^2\omega^2\sin^2\left(\omega t + \phi\right)$



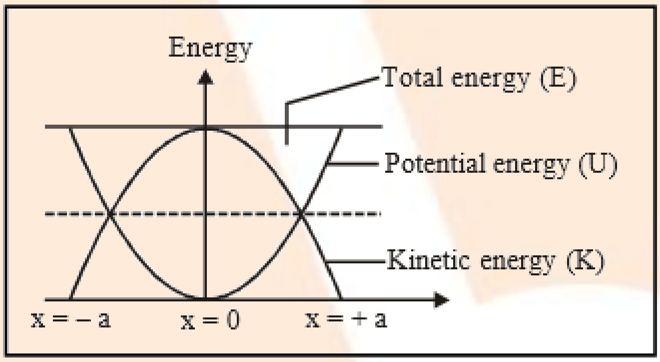
- U is maximum at extremes, given by $U_{max} = \frac{1}{2}kA^2$
- U is minimum at equilibrium position.

Total Energy



- $T.E. = \frac{1}{2}kA^2 = \frac{1}{2}mA^2\omega^2$ and is constant at all instants of time and at all positions.

Energy Position graph



2. TIME PERIOD OF S.H.M

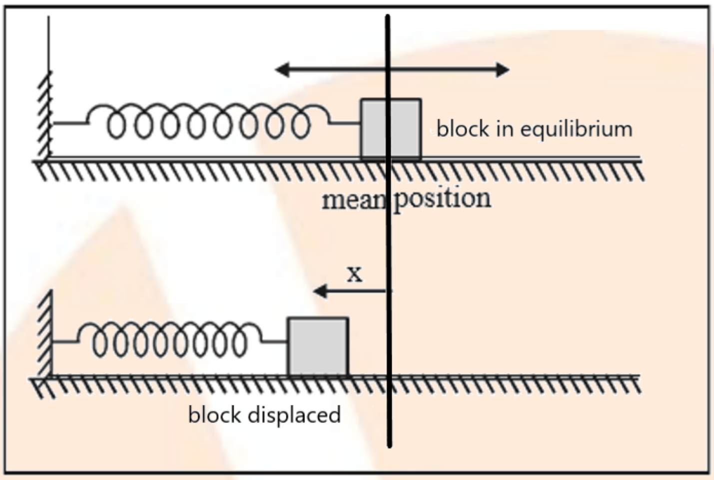
To understand whether a motion is S.H.M. or not and to compute its time period, follow these steps:

a) Locate the equilibrium position mathematically by balancing all the forces on it.
b) Displace the particle by a displacement ‘x’ from the mean position in the probable direction of oscillation.
c) Determine the net force on it and check if it is towards mean position.
d) Try to express net force as a proportional function of its displacement ‘x’.
   - If step (c) and step (d) are proved then it is a simple harmonic motion.
e) Determine k from expression of net force $(F = -kx)$ and find the time period using $T = 2\pi\sqrt{\frac{m}{k}}$.

2.1 Oscillations of a Block Connected to a Spring

a) <u>Horizontal spring</u>:

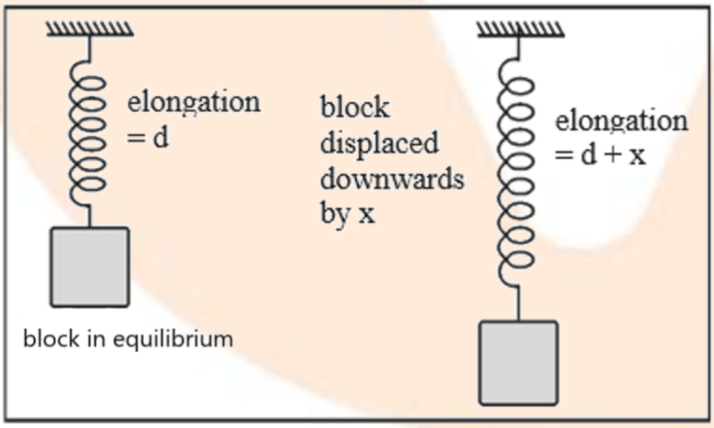
Suppose a block of mass m be placed on a smooth horizontal surface and rigidly connected to spring of force constant K whose other end is permanently fixed.



- Mean position: when spring is at its natural length.
- Time period: $T = 2\pi\sqrt{\frac{m}{k}}$

b) <u>Vertical Spring</u>:

When the spring is suspended vertically from a fixed point and carries the block at its other end as shown, the block will oscillate along the vertical line.



- Mean position: spring in elongated by $d = \frac{mg}{k}$
- Time period: $T = 2\pi\sqrt{\frac{m}{k}}$

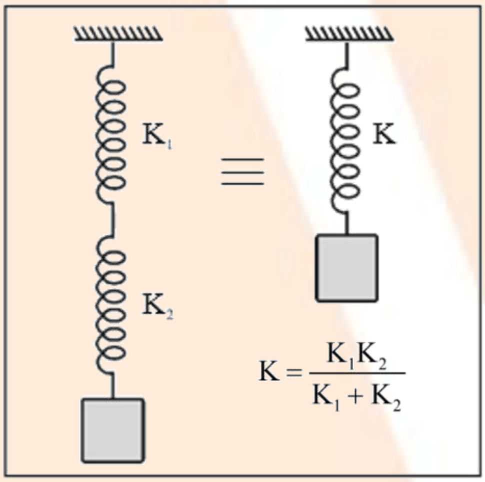
c) <u>Combination of springs</u>:

1. Springs in series:

Consider two springs of force constants K_1 and K_2 respectively, connected in series as shown. They are equivalent to a single spring of force constant K which is given by

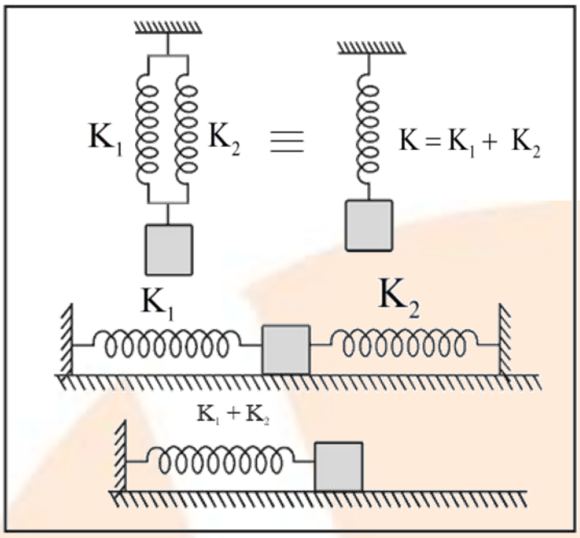
$$\frac{1}{K}=\frac{1}{K_1}+\frac{1}{K_2}$$

$$\Rightarrow K=\frac{K_1K_2}{K_1+K_2}$$



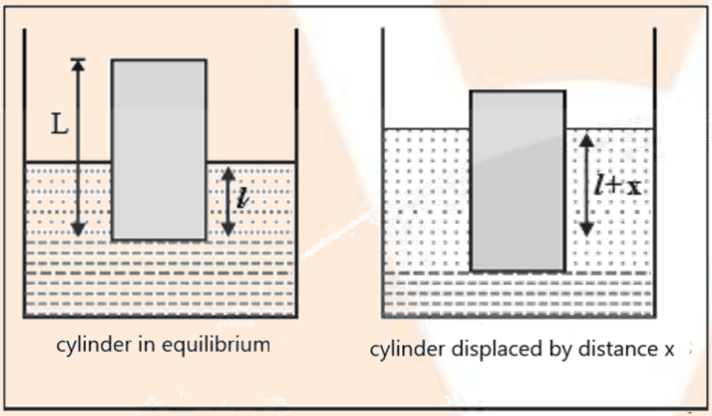
2. Springs in parallel:

For a parallel combination as shown, the effective spring constant is $K=K_1+\ K_2$.



2.2 Oscillation of a cylinder floating in a liquid

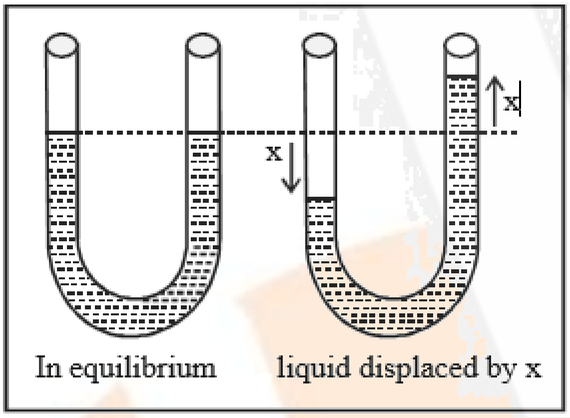
Suppose a cylinder of mass m and density d be floating on the surface of a liquid of density ρ. The total length of cylinder is taken to be L.



- Mean position: cylinder is immersed up to $\ell = \dfrac{Ld}{\rho}$
- Time period: $T = 2\pi\sqrt{\dfrac{Ld}{\rho g}} = 2\pi\sqrt{\dfrac{\ell}{g}}$

2.3 Liquid oscillating in a U–Tube

Consider a liquid column of mass m and density 〉 in a U-tube of area of cross section A.



- Mean position: when height of liquid is the same in both limbs.
- Time period: $T = 2\pi\sqrt{\dfrac{m}{2A\rho g}} = 2\pi\sqrt{\dfrac{L}{2g}}$

2.4 Body oscillation in tunnel along any chord of earth

- Mean position: At the centre of the chord
- Time period: $T = 2\pi\sqrt{\dfrac{R}{g}} = 84.6$ minutes

  where, R is radius of earth

2.5 Angular Oscillations

Instead of straight-line motion, when a particle or centre of mass of a body oscillates on a small arc of circular path, then it is known as angular S.H.M.
For angular S.H.M., torque is given by
$\tau = -k\theta$
where k is a constant and θ is the angular displacement.
$\Rightarrow I\alpha = -k\theta$

where I is the moment of inertia and α is the angular acceleration.

$\Rightarrow T = 2\pi\sqrt{\frac{I}{k}}$, is the time period of oscillations.

2.5.1 Simple pendulum

- Time period: $T = 2\pi\sqrt{\frac{\ell}{g}}$
- Time period of a pendulum in a lift:

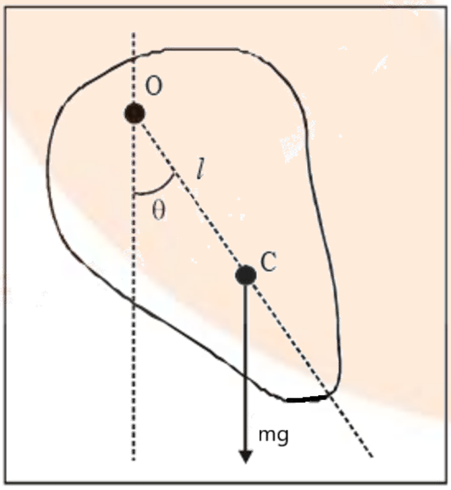
  $T = 2\pi\sqrt{\frac{\ell}{g+a}}$ (If acceleration of lift acts upwards)

  $T = 2\pi\sqrt{\frac{\ell}{g-a}}$ (If acceleration of lift acts downwards)

- For a second's pendulum:
  Time period of second's pendulum is 2s.
  Length of second's pendulum on earth surface $\simeq 1\text{m}$.

2.5.2 Physical pendulum

- Time period: $T = 2\pi\sqrt{\frac{I}{mg\ell}}$



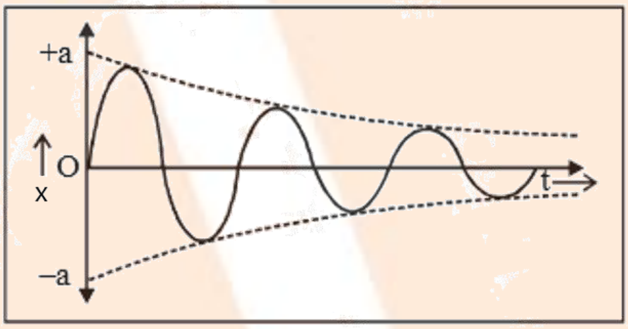
Here,
I refers to the moment of inertia of body about point of suspension.
ℓ refers to the distance of centre of mass of body from point of suspension.

3. DAMPED AND FORCED OSCILLATIONS

1. Damped oscillation:

i. Damped oscillation refers to the type vibration of a body whose amplitude keeps on decreasing with time.

ii. In this type of vibration, the amplitude decreases exponentially because of damping forces like frictional force, viscous force etc.



iii. Because of decrease in amplitude, the energy of the oscillator also keeps on decreasing exponentially.

2. Forced oscillation:

i. Forced oscillation refers to the type of vibration in which a body vibrates under the influence of an external periodic force.

ii. Resonance: When the frequency of external force is the same as the natural frequency of the oscillator, then this state is called as the state of resonance. This equal frequency is known as resonant frequency.

4. WAVES

a) Speed of longitudinal wave

- Speed of longitudinal wave in a medium mathematically represented as

  $$v = \sqrt{\frac{E}{\rho}}$$

  where, E is the modulus of elasticity and ρ is the density of the medium.
- Speed of longitudinal wave in a solid in the form of rod is represented as

$$v=\sqrt{\frac{Y}{\rho}}$$

where, Y is the Young's modulus of the solid and ρ is the density of the solid.

- Speed of longitudinal wave in fluid is represented as

$$v=\sqrt{\frac{B}{\rho}}$$

where, B is the bulk modulus and ρ is the density of the fluid.

b) Newton's formula

- Newton considered that propagation of sound wave in gas is an isothermal process. Clearly, according to him, speed of sound in gas is given by $v=\sqrt{\frac{P}{\rho}}$, where P is the pressure of the gas and ρ is the density of the gas.
- According to the Newton's formula, the speed of sound in air at S.T.P. is 280m/s. However, the experimental value of the speed of sound in air is 332ms^{-1}. Newton could not explain this large difference during his time. In future, his formula was rectified by Laplace.

c) Laplace's correction

- Laplace considered that propagation of sound wave in gas in an adiabatic process. Clearly, according to him, speed of sound in a gas is given by $v=\sqrt{\frac{\gamma P}{\rho}}$, where γ refers to the ratio of specific heats.
- According to Laplace's correction, the speed of sound in air at S.T.P. is 331.3m/s, which agrees fairly well with the experimental values of the speed of sound in air at S.T.P.

5. WAVES TRAVELLING IN OPPOSITE DIRECTIONS

- Consider two waves of equal amplitude and frequency propagating in opposite directions:

$$y_1 = A\sin(kx-\omega t)$$

$$y_2 = A\sin(kx+\omega t)$$

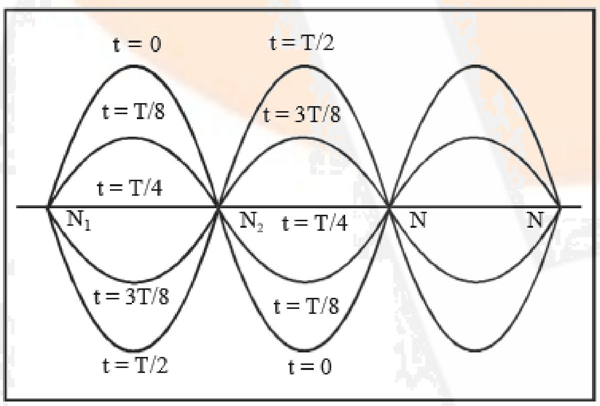
Let them be considered interfering and hence, a standing wave is produced given by,

$$y = y_1 + y_2$$
$$\Rightarrow y = 2A\sin kx \cos\omega t$$

- Clearly, the particle at location x is oscillating in S.H.M. with angular frequency ⌉ and amplitude $2A\sin kx$. Since the amplitude depends on location (x), particles oscillate with different amplitudes.
- **Nodes:**

  Amplitude $= 0$
  $$\Rightarrow 2A\sin kx = 0$$
  $$\Rightarrow x = 0, \frac{\pi}{k}, \frac{2\pi}{k}....$$
  $$\Rightarrow x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, 2\lambda....$$
- **Antinodes:**

  Amplitude is maximum.
  $$\Rightarrow \sin kx = \pm 1$$
  $$\Rightarrow x = \frac{\pi}{2k}, \frac{3\pi}{2k}$$
  $$\Rightarrow x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}$$
- Nodes remain at complete rest whereas antinodes oscillate with maximum amplitude (2A). The points between a node and antinode have amplitude between 0 and 2A.
- Distance between two consecutive nodes (or antinodes) $= \frac{\lambda}{2}$.
- Distance between a node and the next antinode $= \frac{\lambda}{4}$.
- Nodes and antinodes are positioned in an alternative manner.



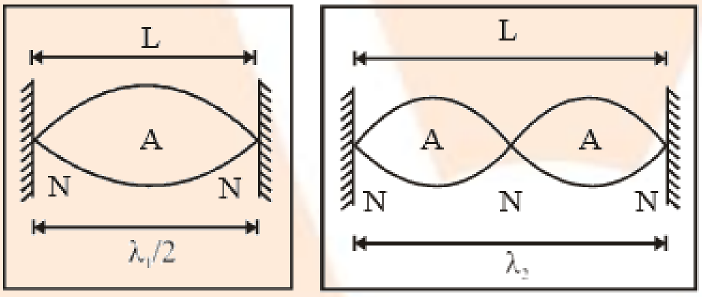
- Figure above suggests that since nodes are at complete rest and thus, they don't transfer energy. In a stationary wave, energy gets no transfer from one point to the other.

5.1 Vibrations in a stretched string

1. Fixed at both ends

- Transverse standing waves with nodes at both ends of the string are formed.
- Clearly, length of string, $\ell = \frac{n\lambda}{2}$ if there are $(n+1)$ nodes and n antinodes.
- Frequency of oscillations is given by

  $$\nu = \frac{v}{\lambda} = \frac{nv}{2\ell}$$



- Fundamental frequency $(x=1)$ or first harmonic:

  $$\nu_0 = \frac{v}{2L}$$

- Second harmonic or first overtone:

  $$\nu = \frac{2v}{2L} = 2\nu_0$$

- The n^{th} multiple of fundamental frequency is called as n^{th} harmonic or $(n-1)^{th}$ overtone.

2. Fixed at one end

- Transverse standing waves with node at fixed end and antinode at open end are formed.
- Clearly, length of string $\ell = (2n-1)\frac{\lambda}{4}$ if there are n nodes and n antinodes.

- Frequency of oscillations is given by

$$\nu = \frac{v}{\lambda} = \frac{(2n-1)v}{4\ell}$$

- Fundamental frequency $(n=1)$ or first harmonic:

$$\nu_0 = \frac{v}{4L}$$

- First overtone or third harmonic.

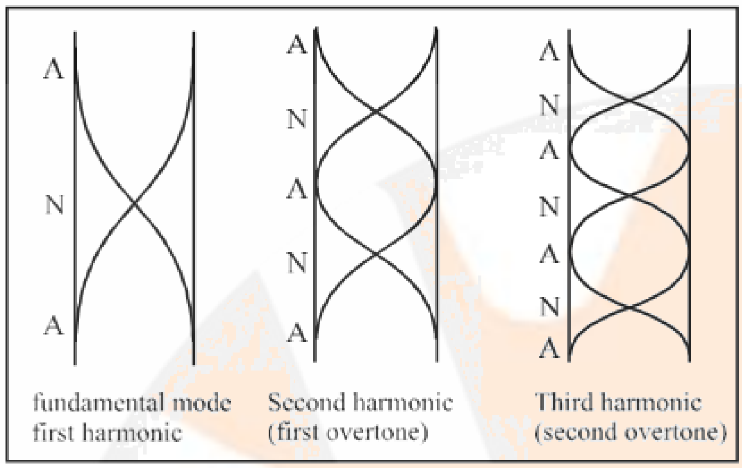
$$\nu = \frac{3v}{4\ell} = 3\nu_0$$

- Only odd harmonics are possible in this case.

5.2 Vibrations in an organ pipe

- **Open Organ pipe (both ends open)**

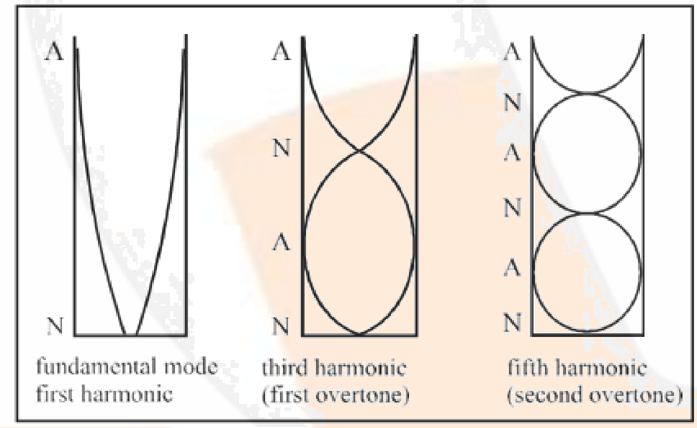
- The open ends of the tube form antinodes as the particles at the open end can oscillate freely.
- When there are $(n+1)$ antinodes in all, length of tube, $\ell = \frac{n\lambda}{2}$.
- Clearly, frequency of oscillations is $\nu = \frac{nv}{2\ell}$.



- **Closed organ pipe (One end closed)**

- The open end forms antinode and closed end forms a node.
- When there are n nodes and n antinodes, $L = (2n-1)\frac{\lambda}{4}$.

- Clearly, frequency of oscillations is $\nu = \frac{v}{\lambda} = \frac{(2n-1)v}{4L}$.



- There are only odd harmonics in a tube closed at one end.

5.3 Waves having different frequencies

- Beats get formed from the superposition of two waves of slightly different frequencies propagating in the same direction. The resultant effect perceived in this case at any fixed position will consist of alternate loud and weak sounds.
- Consider the net effect of two waves of frequencies ν_1 and ν_2 of equal amplitude A at $x = 0$.

$$y_1 = A\sin 2\pi\nu_1 t$$
$$y_2 = A\sin 2\pi\nu_2 t$$
$$\Rightarrow y = y_1 + y_2$$
$$\Rightarrow y = A(\sin 2\pi\nu_1 t + \sin 2\pi\nu_2 t)$$
$$\Rightarrow y = [2A\cos\pi(\nu_1 - \nu_2)t]\sin\pi(\nu_1 + \nu_2)t$$

Clearly, the resultant wave can be denoted as a travelling wave whose frequency is

$\left(\frac{\nu_1 + \nu_2}{2}\right)$ and amplitude is $2A\cos\pi(\nu_1 - \nu_2)t$.

Since the amplitude term contains t, the amplitude varies periodically with time.

For loud sounds:

Net amplitude $= \pm 2A$

$$\Rightarrow \cos\pi(\nu_1 - \nu_2)t = \pm 1$$

$$\Rightarrow \pi(\nu_1 - \nu_2)t = 0, \pi, 2\pi, 3\pi, ...$$

$$\Rightarrow t = 0, \frac{1}{\nu_1 - \nu_2}, \frac{2}{\nu_1 - \nu_2}, ...$$

Thus, the interval between two loud sounds represented as $\frac{1}{\nu_1 - \nu_2}$.

$\Rightarrow$ The number of loud sounds per second $= \nu_1 - \nu_2$.

$\Rightarrow$ Beat per second $= \nu_1 - \nu_2$.

Here, it is to be noticed that $\nu_1 - \nu_2$ should be small (0–16Hz) in order to distinguish sound variations.

Note:

- Filing a tuning fork increases its frequency of vibration whereas loading a tuning fork decreases its frequency of vibration.

6. DOPPLER EFFECT

Doppler's effect suggests that whenever there is a relative motion between a source of sound and a listener, the apparent frequency of sound heard by the listener is different from the actual frequency of sound emitted by the source. Mathematically, apparent frequency is given by $\nu' = \frac{v - v_L}{v - v_S} \times \nu$.

Sign convention:

- All velocities along the direction S to L are taken as positive and all velocities along the direction L to S are taken as negative.
- When the motion is along some other direction, the components of velocity of source and listener along the line joining the source and listener would be considered.

Special Cases:

a) If the source is moving towards the listener but the listener is at rest, then v_S is positive and $v_L = 0$ (figure a). Clearly,

$$\nu' = \frac{v}{v - v_S} \times \nu \text{ i.e., } \nu' > \nu$$

b) If the source is moving away from the listener, but the listener is at rest, then ν_S is negative and $\nu_L = 0$ (figure b). Clearly,

$$\nu' = \frac{\nu}{\nu - (-\nu_S)} \times \nu = \frac{\nu}{\nu + \nu_S}\nu \text{ i.e., } \nu' < \nu$$

c) If the source is at rest and listener is moving away from the source, the $\nu_S = 0$ and ν_L is positive (figure c). Clearly,

$$\nu' = \frac{(\nu - \nu_L)}{\nu}\nu \text{ i.e., } \nu' < \nu$$

d) If the source is at rest and listener is moving towards the source, then $\nu_S = 0$ and ν_L is negative (figure d). Clearly,

$$\nu' = \frac{\nu - (-\nu_L)}{\nu - \nu_S}\nu = \frac{\nu + \nu_L}{\nu}\nu \text{ i.e., } \nu' > \nu$$

e) If the source and listener are approaching each other, then ν_S is positive and ν_L is negative (figure e). Clearly,

$$\nu' = \frac{\nu - (-\nu_L)}{\nu - \nu_S}\nu = \frac{\nu + \nu_L}{\nu - \nu_S}\nu \text{ i.e., } \nu' > \nu$$

f) If the source and listener are moving away from each other, then ν_S is negative and ν_L is positive (figure f). Clearly,

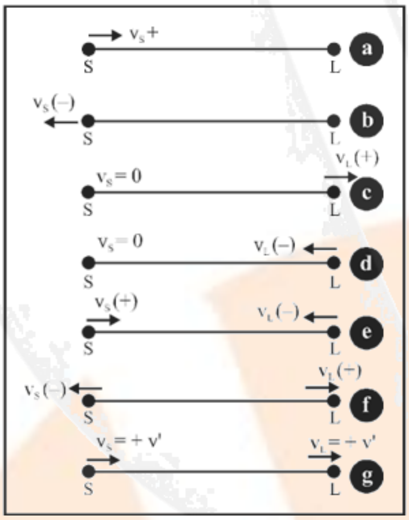
$$\nu' = \frac{\nu - \nu_L}{\nu - (-\nu_S)}\nu = \frac{\nu - \nu_L}{\nu + \nu_S}\nu \text{ i.e., } \nu' < \nu$$

g) If the source and listener are both in motion in the same direction and with same velocity, then $\nu_S = \nu_L = \nu'$ (figure g). Clearly,

$$\nu' = \frac{(\nu - \nu')}{(\nu - \nu')} \text{ i.e., } \nu' = \nu$$

This suggests that there is no change in the frequency of sound perceived by the listener.

Apparent wavelength perceived by the observer can be given as $\lambda' = \frac{\nu - \nu_S}{\nu}$.



Note: In case the medium is also in motion, the speed of sound with respect to the ground is also considered. i.e., $\vec{v} + \vec{v}_m$.

7. CHARACTERISTICS OF SOUND

- **Loudness** of sound is also known as level of intensity of sound.
  In decibels, the loudness of a sound of intensity I is represented by
  $$L = 10\log_{10}\left(\frac{I}{I_0}\right)$$
  where, $I_0 = 10^{-12}\, W/m^2$
- **Pitch** of a sound depends on its frequency. Higher the frequency of sound, higher would be its pitch and shriller would it be.