Revision Notes

Class 11 physics

Chapter 2- Units and Measurements

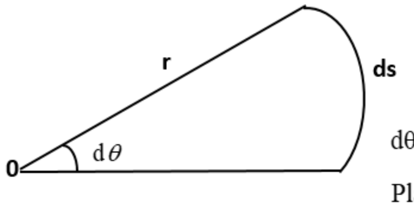
Units

A unit can be defined as an internationally accepted standard for measuring quantities.

- Measurement has been included of a numeric quantity along with a specific unit.
- The units in the case of base quantities (such as length, mass etc.) are defined as Fundamental units.
- Derived units are the units which are combination of fundamental units.
- Fundamental and Derived units constitute together as a System of Units.
- Internationally accepted system of units can be defined as Systèmе Internationale d' Unites (This is how International system of Units represented in French) or SI. In 1971, it was produced and recommended by General Conference on Weights and Measures.
- The table shown below is the list of 7 base units mentioned by SI.

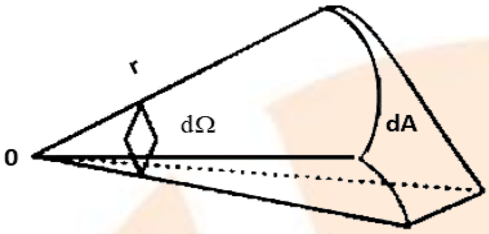
There are two units along with it. They are, radian or rad (unit for plane angle) and steradian or sr (unit for solid angle). Both of these are dimensionless.

Base Quantity	Name	Symbol
Length	metre	m
Mass	kilogram	kg
Time	second	s
Electric Current	ampere	A
Thermodynamic Temperature	kelvin	K
Amount of Substance	mole	mol
Luminous intensity	candela	cd



$d\theta = \frac{ds}{r}$ radian

Plane angle.Unit-Radian

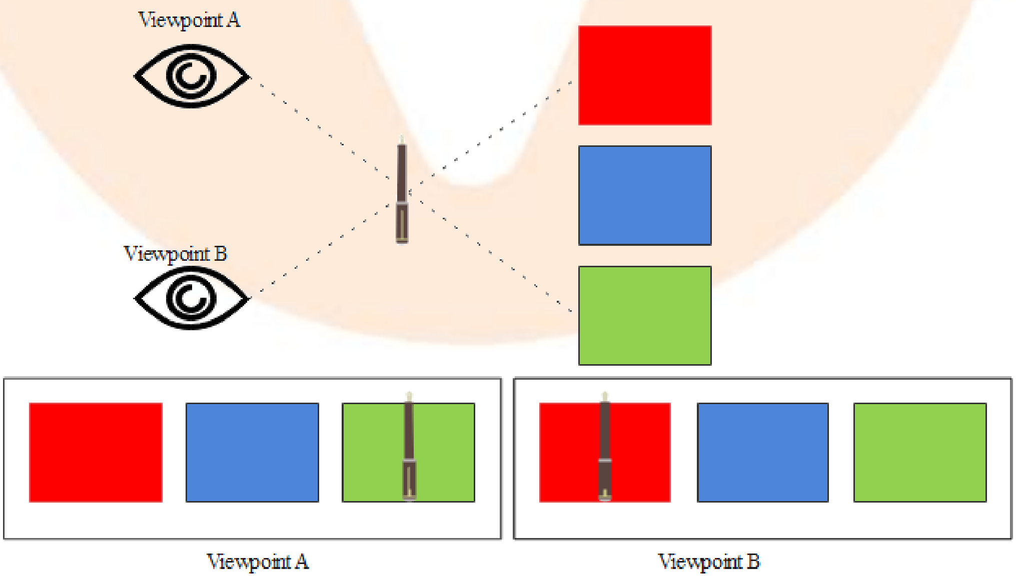


$d\Omega = \frac{dA}{r^2}$ steradian

Solid angle.Unit-Steradian

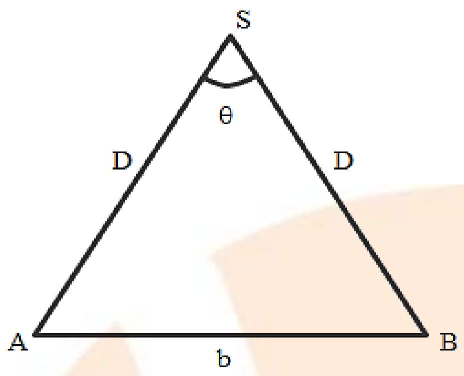
2.3.1. Parallax Method-Measurement of large distances

- Parallax can be defined as a displacement or difference in the apparent position of a body viewed along two various lines of sight, and is calculated by the angle or semi-angle of inclination between those two lines. Basis is called as the distance between the two viewpoints.



Parallax. From viewpoint A the pen appears over green box while from viewpoint B the pen appears over red box

Calculating the distance of a planet using parallax method



S-position of planet
D-distance from the two viewpoints or observatories
q-parallax or parallactic angle

For far away planet, $b/D \ll 1$
Hence AB is taken as arc of length b and D is radius with S as center.
So, b=Dq or D=b/q

Parallax method to determine distance of a planet

In the similar way,

$$\alpha = \frac{d}{D}$$

Where α be the planet's angular size (angle subtended by d at earth) and d will be the diameter of the planet. If two diametrically opposite points of the planet are viewed, then α be the angle between the direction of the telescope.

2.3.2. Measuring very small distances

For measuring the distances as low as size of a molecule, electron microscopes will be used. These will include electrons beams controlled by electric and magnetic fields.

- Electron microscopes will be having a resolution of 0.6 Å or Angstroms.
- Electron microscopes are used for resolving atoms and molecules when we use a tunnelling microscopy, it will be possible for estimating size of molecule. Calculating size of molecule of Oleic acid. Oleic acid will be a soapy liquid with large molecular size of the order of 10^{-9} m. The following steps are used in determining the size of molecule:
- Dissolving 1 cm^3 of oleic acid in alcohol for producing a
- solution of 20 cm^3.Consider 1 cm^3 of above solution and using

alcohol dilute it to a concentration of $20\ cm^3$. So, the concentration of oleic acid in the solution will become $\left(\frac{1}{20\times 20}\right)$ cm^3 of oleic acid/cm^3 of solution.

- Then add lycopodium powder on the surface of water in a trough and keep one drop of above solution. In a circular molecular thick film, the oleic acid in the solution will spread over water.
- Calculate the diameter of the above circular film by the use of below calculations.
- When n –Number of drops of solution in water, t – Thickness of the film, V –Volume of each drop, A – Area of the film.

  Total volume of n drops of solution $=nVcm^3$

  The amount of Oleic acid in this solution $=nV(120\times 20)cm^3$

  $$\text{Thickness of the film} = t = \frac{\text{Volume of the film}}{\text{Area of the film}}$$

  $$t=\frac{nV}{20\times 20A}cm$$

Special Length units

Unit name	Unit Symbol	Value in meters
fermi	f	10^{-15} m
angstrom	Å	10^{-10} m
Astronomical unit (average distance of sun from earth)	AU	1.496×10^{11} m
light year(distance travelled by light in 1 year with velocity $3\times 10^8\ ms^{-1}$)	ly	9.46×10^{11} m
parsec(distance at which average radius of orbit of the earth subtends an angle of 1 arc second)	pc	3.08×10^{16} m

Measurement of Mass

Mass can be defined usually in terms of kg but unified atomic mass unit (u) will be used for atoms and molecules.

$1\text{ u}=\frac{1}{12}$ of the mass of an atom of isotope of carbon 12 which will be included of the mass of electrons (1.66×10^{-27} kg).Mass of planets is measured by the use of gravitational methods and mass of atomic particles are measured by the usage of mass spectrograph (radius of trajectory will be proportional to mass of charged particle which is in motion in uniform electric and magnetic field), apart from using balances for normal weights.

Range of Mass

Object	Mass(kg)
Electron	10^{-30}
Proton	10^{-27}
Red blood cell	10^{-13}
Dust particle	10^{-9}
Rain drop	10^{-6}
Mosquito	10^{-5}
Grapes	10^{-3}
Human	10^{2}
Automobile	10^{3}
Boeing 747 aircraft	10^{8}
Moon	10^{23}
Earth	10^{25}
Sun	10^{30}
Milky way Galaxy	10^{41}
Observable Universe	10^{55}

Measurement of Time

Time has been calculated by the use of a clock. Atomic standard of time being now used, measured by the Cesium or Atomic clock, as a standard.

- A second will be equivalent to 9,192,631,770 vibrations of radiation from the transition between two hyperfine levels of an atom of cesium-133 in a Cesium clock.

- Cesium clock will be working on the vibration of Cesium atom which is identical to vibrations of quartz crystal in a quartz wristwatch and balance wheel in normal wristwatch.
- National standard time and frequency will be maintained by 4 atomic clocks. Indian standard time will be maintained by a Cesium clock at National Physical Laboratory (NPL), New Delhi.
- Cesium clocks will be perfectly accurate and the uncertainty will be very low 1 part in 1013 which means not more than $3\ \mu s$ will be lost or gained in a year.

Range of Time

Event	Time Interval (s)
Life span of most unstable particle	10^{-24}
Period of x-rays	10^{-19}
Period of light wave	10^{-15}
Period of radio wave	10^{-6}
Period of sound wave	10^{-3}
Wink on an eye	10^{-1}
Time of travel of light from moon to earth	10^{0}
Time of travel of light from sun to earth	10^{2}
Rotation period of the earth	10^{5}
Revolution period of the earth	10^{7}
Average human life span	10^{9}
Age of Egyptian pyramids	10^{11}
Time since dinosaur extinction	10^{15}
Age of Universe	10^{17}

Accuracy and Precision of Instruments

- Any uncertainty which is formed from calculation by a measuring instrument can be defined as an error. This can be categorised as systematic or random.
- The resolution of the measured value to the true value can be defined as accuracy of a measurement.
- The resolution of a numerous of measurements of an identical quantity under same conditions can be called as precision.
- Till 1 (less precise) and 2 (more precise) decimal places in the same order, has been used when the true value of a specific length is 3.678 cm and two instruments with various resolutions. When first measures the length as 3.5 and the second as 3.38 then the first will be having more accuracy but precision will be less while the second will be having less accuracy and more precision.

Types of Errors- Systematic Errors

Systematic errors are the errors which can either be positive or negative. The following types are:

1. Instrumental errors: which is arouse from calibration error or imperfect design in the instrument. Zero error in a weighing scale, Worn off scale are a few examples of instrument errors.

2. Imperfections in experimental techniques when the technique is not correct (measurement of temperature of human body by keeping thermometer under armpit resulting in lower temperature than actual can be considered as an example) and because of the external conditions such as temperature, wind, humidity, and these kinds of errors happen.

3. Personal errors: Errors happening because of the human carelessness, lack of proper setting, taking down incorrect reading are defined as personal errors.

The removal of these errors will be:

- By taking proper instrument and properly calibrating it.
- By experimenting under correct atmospheric conditions and techniques.
- Avoiding human bias as far as possible.

Random Errors

Errors which is happening at random with respect to sign and size are defined as Random errors.

- These kind of errors happens because of the unpredictable fluctuations in experimental conditions such as temperature, voltage supply, mechanical vibrations, personal errors etc.

Least Count Error

The smallest value which is measurable by the use of a measuring instrument is called its least count. Least count error is the error related with the least count of the instrument.

- Least count errors is minimizable by the usage of equipments of higher precision/resolution and improving experimental techniques (take several readings of a measurement and then calculate a mean).

Errors in a series of Measurements.

Assume that the values got in several measurement are $a_1, a_2, a_3....a_n$.

Arithmetic mean,

$$a_{mean} = \frac{(a_1+a_2+a_3+...+a_n)}{n}$$

$$a_{mean} = \sum_{i=1}^{n} \frac{a_i}{n}$$

- Absolute Error can be defined as the magnitude of the difference between the true value of the quantity and absolute error of the measurement can be defined as the individual measurement value. It is represented as $|\Delta a|$ (or Mod of Delta a). The mod value will be positive always even if Δa is negative. The individual errors will be:

$$\Delta a_1 = a_{mean} - a_1$$
$$\Delta a_2 = a_{mean} - a_2$$
....
....
$$\Delta a_n = a_{mean} - a_n$$

- Mean absolute error can be explained as the arithmetic mean of all absolute errors. It has been represented as Δa_{mean}.

  $\Delta a_{mean} = |\Delta a_1| + |\Delta a_2| + |\Delta a_3| + ... + |\Delta a_n|$

  $$\Delta a_{mean} = \sum_{i=1}^{n} \frac{|\Delta a_i|}{n}$$

In the case of every single measurement, the value of 'a' is always in the range $a_{mean} \pm \Delta a_{mean}$

So, $a = a_{mean} \pm \Delta a_{mean}$

Or, $a_{mean} - \Delta a_{mean} \le a \le a_{mean} + \Delta a_{mean}$

- Relative Error can be defined as the mean absolute error divided by the mean value of the quantity measured.

  $$\text{Relative Error} = \frac{\Delta a_{mean}}{a_{mean}}$$

- Percentage Error can be defined as the relative error expressed in percentage. It is denoted by δa. $\delta a = \frac{\Delta a_{mean}}{a_{mean}} \times 100$

Combinations of Errors

When a quantity is dependent on two or more other quantities, the combination of errors in the two quantities will be helping for determining and predicting the errors in the resultant quantity. There

Criteria	Sum or Difference	Product	Raised to Power
Resultant value Z	$Z = A \pm B$	$Z = AB$	$Z = A^k$
Result with error	$Z \pm \Delta Z = (A \pm \Delta A) + (B \pm \Delta B)$	$Z \pm \Delta Z = (A \pm \Delta A)(B \pm \Delta B)$	$Z \pm \Delta Z = (A \pm \Delta A)^k$
Resultanat error	$\pm \Delta Z = \pm \Delta A + \Delta B$	$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} \pm \frac{\Delta B}{B}$	
range			
Maximum error	$\Delta Z = \Delta A + \Delta B$	$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$	$\frac{\Delta Z}{Z} = k\left(\frac{\Delta A}{A}\right)$

Error	Sum of absolute error	Sum of relative errors	K multiplied by relative error

Consider two quantities A and B have values as $A \pm \Delta A$ and $B \pm \Delta B$. Let Z be the result and ΔZ is the error because of the combination of A and B.

Significant Figures

Every measurement gives us an output in a number that is included of reliable digits and uncertain digits. Reliable digits added with the first uncertain digit can be defined as significant digits or significant figures. This is representing the precision of measurement which is dependent on least count of instrument used for measurement.The period of oscillation of a pendulum is 1.62 s can be taken as an example. Here 1 and 6 will be the reliable and 2 is uncertain. Hence, the measured value will be having three significant figures.

Rules for the determination of number of significant figures

- All non-zero digits will be significant.
- Irrespective of decimal place, all zeros between two non-zero digits will be significant irrespective of decimal place.
- Zeroes before non-zero digits and after decimal are not considered as significant, for a value less than 1. Zero present before decimal place in case of these number will be insignificant always.
- Trailing zeroes in case of a number without any decimal place will be insignificant.
- Trailing zeroes in case of a number with decimal place will be significant.

Cautions for removing ambiguities in calculating number of significant figures

- Variation of units will not change number of significant digits. As an example,

$$4.700 \text{ m} = 470.0 \text{ cm}$$
$$= 4700 \text{ mm}$$

Here, first two quantities are having 4 but third quantity is having 2 significant figures.

- Make use of scientific notation for reporting measurements. Numbers must be shown in powers of 10 such as $a\times10b$ where b is defined as order of magnitude. Example,
  $4.700 \text{ m} = 4.700 \times 10^2 \text{ cm}$
  $= 4.700 \times 10^3 \text{ mm}$
  $= 4.700 \times 10^{-3} \text{ km}$
  Here, as power of 10 is being irrelevant, number of significant figures will be 4.

- Multiplying or dividing exact numbers will be giving infinite number of significant digits. Example, $\text{radius} = \frac{\text{diameter}}{2}$. In this case, 2 can be represented as 2, 2.0, 2.00, 2.000 and so on.

Rules for Arithmetic operation with Significant Figures

Type	Multiplication or Division	Addition or Subtraction
Rule	The end result must retain as many significant figures as there in the initial number with the least number of significant digits.	The end result must have as many decimal places similar way as in the original number with the least decimal places.
Example	$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$ Assume $\text{Mass} = 4.237 \text{ g}$ (4 significant figures) and $\text{Volume} = 2.51 \text{ cm}^3$ (3 significant figures)	Addition of 436.32 (2 digits after decimal), 227.2 (1 digit after decimal) and .301 (3 digits after decimal) is= 663.821

	Density= $\frac{4.237\ g}{2.51\ cm^3}$ $=1.68804\ gcm^{-3}$ $=1.69\ gcm^{-3}$ (3 significant figures)	As 227.2 is precise up to only 1 decimal place, Therefore, the end result should be 663.8.

Rules for Rounding off the uncertain digits

Rounding off will be essential for reducing the number of insignificant figures to hold to the rules of arithmetic operation with significant figures.

Rule Number	Insignificant digit	Preceding digit	Example (rounding off to two decimal places)
1	Insignificant digit to be dropped being more than 5	Preceding digit is raised by 1.	Number– 3.137 Result –3.14
2	Insignificant digit to be dropped being less than 5	Preceding digit is left unchanged.	Number– 3.132 Result –3.13
3	Insignificant digit to be dropped being equal to 5	When preceding digit is even, it is left unchanged.	Number– 3.125 Result –3.12
4	Insignificant digit to be dropped being equal to 5	When preceding digit is odd, it is raised by 1.	Number– 3.135 Result –3.14

Rules for the determination of uncertainty in the results of arithmetic calculations

For calculating the uncertainty, below process must be used.

- Do summation of a lowest amount of uncertainty in the original numbers. Example uncertainty for 3.2 will be ± 0.1 and for 3.22 will be ± 0.01.
- Find out these in percentage also.
- The uncertainties get multiplied/divided/added/subtracted after the calculations.
- In the uncertainty, round off the decimal place for obtaining the end uncertainty result.

  For example, for a rectangle,

  Suppose length, $l=16.2\text{ cm}$ and

  breadth, $b=10.1\text{ cm}$

  After that, take $l=16.2\pm 0.1\text{ cm}$ or $l=16.2\text{ cm}\pm 0.6\ \%$ and

  breadth $=10.1\pm 0.1\text{ cm}$ or $10.1\text{ cm}\pm 1\ \%$

  When we multiply,

  $\text{area}=\text{length}\times\text{breadth}=163.62\text{ cm}^2\pm 1.6\ \%$

  Or $163.62\pm 2.6\text{ cm}^2$

  Hence after rounding off, $\text{area}=164\pm 3\text{ cm}^2$.

  Therefore 3 cm^2 will be the uncertainty or the error in estimation.

Rules

1. In the case of a set experimental data of 'n' significant figures, the result must be accurate to 'n' significant figures or less (only in case of subtraction).

For example $12.9-7.06=5.84$ or 5.8 (when we round off to least number of decimal places of original number).

2. The relative error of a value of number mentioned to significant figures will be dependent on n and on the number itself.

As an example, say accuracy for two numbers 1.02 and 9.89 be ± 0.01.
But relative errors are:

For 1.02, $\left(\frac{\pm 0.01}{1.02}\right)\times 100\ \% = \pm 1\ \%$

For 9.89, $\left(\frac{\pm 0.01}{9.89}\right)\times 100\ \% = \pm 0.1\ \%$

Therefore, the relative error will be dependent upon number itself.

3. The results in the intermediate step of a multi-step computation should be found to have one significant figure more in all the measurement than the number of digits in the least precise measurement.

For example: $\frac{1}{9.58} = 0.1044$

Now, $\frac{1}{0.104} = 9.56$ and $\frac{1}{0.1044} = 9.58$

Therefore, taking one extra digit will be providing more precise outputs and reduces rounding off errors.

Dimensions of a Physical Quantity

The powers (exponents) to which base quantities are raised to represent that quantity can be defined as dimensions of a physical quantity. They are figured as the square brackets around the quantity.

- Dimensions of the 7 base quantities has been considered as – Length [L], time [T], Mass [M], thermodynamic temperature [K], luminous intensity [cd], electric current [A] and amount of substance [mol].
  For example,

  $$\text{Volume} = \text{Length} \times \text{Breadth} \times \text{Height}$$

  $$= [L]\times[L]\times[L] = [L]^3$$

  $$\text{Force} = \text{Mass} \times \text{Acceleration}$$

  $$= \frac{[M][L]}{[T]^2} = [M][L][T]^{-2}$$

- The other dimensions for a quantity will be always 0. As an example, in the case of volume only length has 3 dimensions but the mass, time etc will be having 0 dimensions. Zero dimension is shown by superscript 0 like $\left[M^0\right]$.Dimensions will not affect the magnitude of a quantity Dimensional formula an Dimensional Equation

The expression which is representing how and which of the base quantities represent the dimensions of a physical quantity is defined as Dimensional Formula.

An equation we got after equating a physical quantity with its dimensional formula is a Dimensional Equation.

Physical Quantity	Dimensional Formula	Dimensional Equation
Volume	$\left[M^0L^3T^0\right]$	$[V]=\left[M^0L^3T^0\right]$
Speed	$\left[M^0LT^{-1}\right]$	$[v]=\left[M^0LT^{-1}\right]$
Force	$\left[MLT^{-2}\right]$	$[F]=\left[MLT^{-2}\right]$
Mass Density	$\left[ML^{-3}T^0\right]$	$[\rho]=\left[ML^{-3}T^0\right]$

Dimensional Analysis

- The physical quantities which is having similar dimensions only can be added and subtracted. This can be named as principle of homogeneity of dimensions.
- Dimensions are multipliable and can be cancelled as normal algebraic methods.
- Quantities on both sides should always have identical dimensions, in mathematical equations.
- Arguments of special functions such as trigonometric, logarithmic and ratio of similar physical quantities will be dimensionless.
- Equations will be uncertain to the extent of dimensionless quantities.

  As an example, say Distance = Speed x Time. In Dimension terms, $[L]=\left[LT^{-1}\right]\times[T]$

As the dimensions can be cancelled like we do in algebra, dimension $[T]$ will get cancelled and the equation will be $[L]=[L]$.

Applications of Dimensional Analysis

When we check the Dimensional Consistency of equations

- A dimensionally correct equation should be having identical dimensions on both the sides of the equation.
- There is no need for a dimensionally correct equation to be a correct equation but a dimensionally incorrect equation will be always incorrect. Dimensional validity can be tested but not calculate correct relationship between the physical quantities.

  Example, $x=x_0+v_0t+\left(\frac{1}{2}\right)at^2$

  Or, Dimensionally, $[L]=[L]+\left[LT^{-1}\right][T]+\left[LT^{-2}\right]\left[T^2\right]$

  Where, x be the distance travelled in time t,

  x_0 – starting position,

  v_0 - initial velocity,

  a – uniform acceleration.

  Dimensions on both sides will be [L] because [T] get cancelled out. Therefore this will be dimensionally correct equation.

Deducing relation among physical quantities

- For deducing a relation among physical quantities, we must know the dependence of one quantity over others (or independent variables) and assume it as product type of dependence.
- Dimensionless constants will not be obtainable by the use of this method.

  We can take an example,

  $T=kl^xg^ym^z$

  Or,

  $$\left[L^0M^0T^1\right]=\left[L^1\right]^x\left[L^1T^{-2}\right]^y\left[M^1\right]^z$$
  $$=\left[L^{x+y}T^{-2y}M^z\right]$$

  This means that, $x+y=0,-2y=1$ and $z=0$. So $x=\frac{1}{2},y=-\frac{1}{2}$ and $z=0$.

Hence the original equation will be reduced to $T=k\sqrt{\frac{l}{g}}$.